The University of Texas at Dallas

Erik Jonsson School
PhoTEC

REVIEW OF OPTICS

Notes prepared for EE 6334

by

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OPTICS AND PHOTONICS MARKETS

- Aerospace
  - ▷ Sensors, displays, targeting, and communications
- Industrial
  - ▷ Welding, machining & writing
  - ▷ Imaging and lithography
- Medical
  - ▷ Surgery, photodynamic therapy, ...
  - ▷ Monitoring
  - ▷ Telemedicine (requires broadband communications)
- Business and entertainment
  - ▷ Imaging and projection (increasingly digital, but requires analog optics)
  - ▷ Information storage and retrieval (DVDs, ...)
- Telecommunications

PROFESSIONAL SOCIETIES AND CORPORATIONS

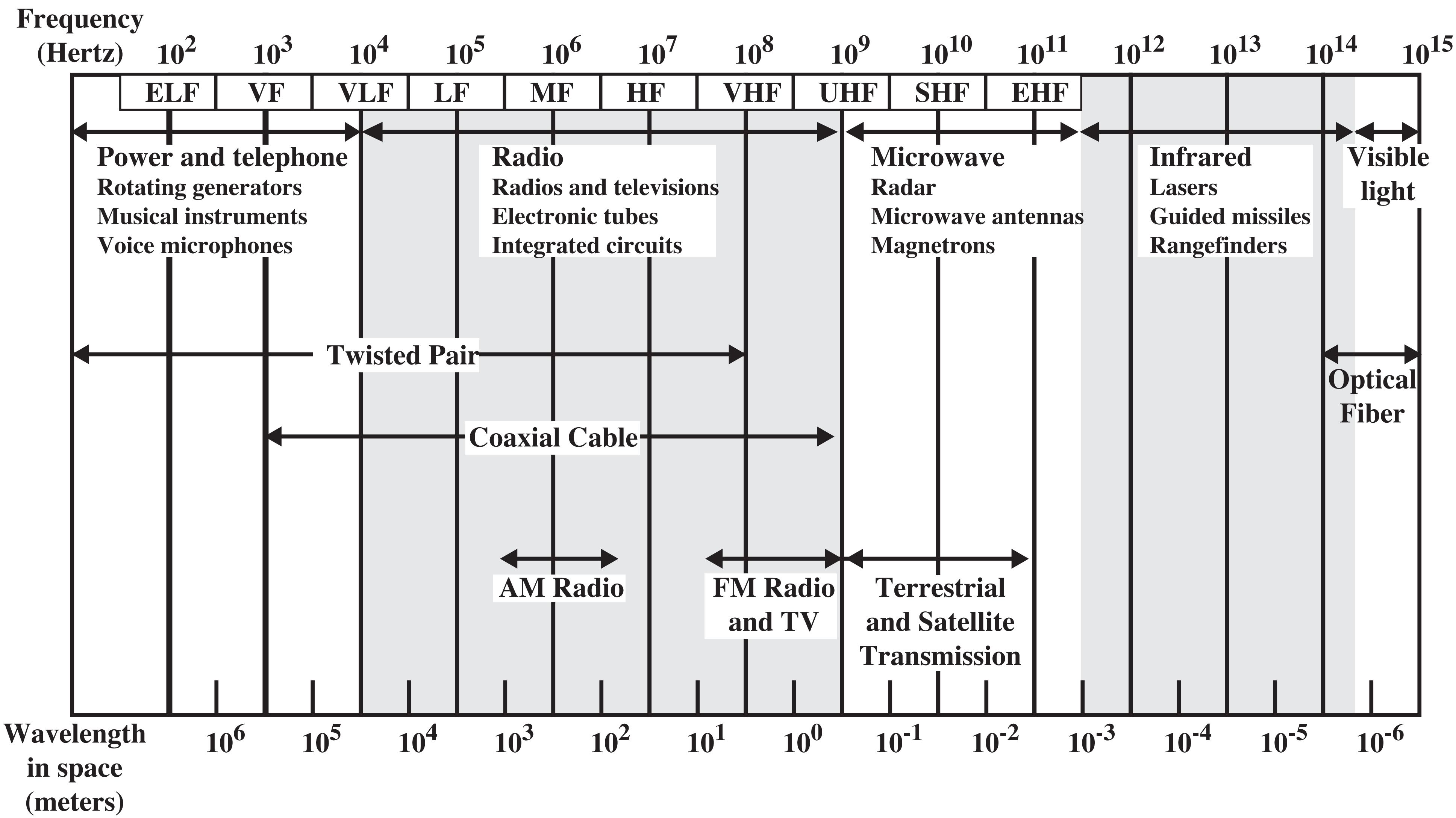
- Optical Society of America (OSA)
  - ▷ Oldest optics/photonics society in North America
  - ▷ Covers all fields of optics, from human vision to optical physics
  - ▷ Peer-reviewed journals include Journal of the Optical Society of America, Applied Optics, Optics Letters, Journal of Lightwave Technology (co-sponsored with IEEE-LEOS), Journal of Optical Networking, Optics Express
- IEEE Lasers and Electro-Optics Society (IEEE-LEOS)
  - ▷ Journal of Quantum Electronics, Photonics Technology Letters, Journal of Special Topics in Quantum Electronics
  - ▷ Co-sponsors the Optical Fiber Communication Conference (OFC) and the Conference on Lasers and Electro-Optics (CLEO) with OSA
- SPIE
  - ▷ Not-for-profit corporation
  - ▷ Organizes many conferences and publishes proceedings

OUTLINE OF OPTICS REVIEW (1)

- Ray optics
  - ▷ Snell's laws of reflection and refraction at dielectric interfaces
  - ▷ Total internal reflection
  - ▷ When can one use ray optics instead of wave optics?
  - ▷ Applications of ray optics will include slab and fiber waveguides
- Plane electromagnetic waves
  - ▷ Polarization
  - ▷ Refractive index
  - ▷ Phase-velocity dispersion
- Reflection coefficients of plane waves at dielectric interfaces
  - ▷ Fresnel's formulas
  - ▷ Brewster's angle
  - ▷ Applications will include polarizing prisms

OUTLINE OF OPTICS REVIEW (2)

- Imaging
  - ▷ Refraction at spherical surfaces; thin lenses
  - ▷ Numerical aperture
- Interference
  - ▷ 2-slit interference (Young's fringes)
  - ▷ Multiple-slit interference
  - ▷ Interferometers
  - ▷ Applications will include modulators, multiplexers and demultiplexers
- Diffraction
  - ▷ Single slit
  - ▷ Multiple slits and diffraction gratings
  - ▷ Applications will include pulse compression



ELF = Extremely low frequency
VF = Voice frequency
VLF = Very low frequency
LF = Low frequency
MF = Medium frequency
HF = High frequency
VHF = Very high frequency
UHF = Ultrahigh frequency
SHF = Superhigh frequency
EHF = Extremely high frequency

Electromagnetic Spectrum for Telecommunications

William Stallings, ***Data and Computer Communications***, 6th Edition

PHASE VELOCITY

- The **phase velocity** of a monochromatic wave is the ratio of its angular frequency to its propagation constant
  - ▷ For example, $E(t) = \cos(\omega t - \beta z)$

    is a monochromatic wave
  - ▷ The angular frequency is

    $$\omega = 2\pi f$$

    where f is the frequency in Hz
  - ▷ The propagation constant is

    $$\beta = \frac{2\pi}{\lambda}$$

    where λ is the wavelength in the medium (not in vacuum)
  - ▷ The phase velocity is

    $$v = \frac{\omega}{\beta} = \lambda f$$

INDEX OF REFRACTION

- The **index of refraction** of a medium is the ratio of the velocity of light in vacuum (c) to the phase velocity in the medium (v):

$$n = \frac{c}{v} = \sqrt{\mu_r \epsilon_r}$$

▷ The refractive index n is a function of wavelength λ (or frequency f) because the phase velocity v depends on λ

▷ The dependence of n on λ gives rise to **phase-velocity dispersion**

▷ **Normal dispersion**: If $\lambda_1 > \lambda_2$, then $v_1 > v_2$ and therefore

$$n(\lambda_1) < n(\lambda_2)$$

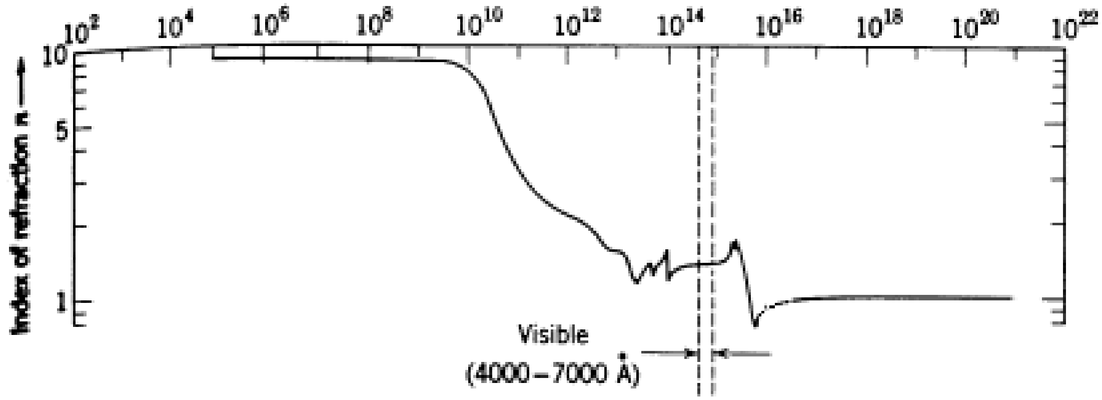
A way to remember this: "Red cars go faster" ☺

▷ **Anomalous dispersion**: If $\lambda_1 > \lambda_2$, then $v_1 < v_2$ and therefore

$$n(\lambda_1) > n(\lambda_2)$$

Usually anomalous dispersion occurs only very close to the frequency of a transition between energy levels in the medium, or as a result of dipolar relaxation (as in water)

REFRACTIVE INDEX OF WATER



- For water, "normal" dispersion,

$$f_1 < f_2 \Rightarrow n(f_1) < n(f_2),$$

holds in the visible spectrum (400–700 nm, or 4.3×10^{14}–7.5×10^{14} Hz), but is not the rule in much of the rest of the electromagnetic spectrum

- ▷ Anomalous dispersion between $\sim 10^{10}$ Hz and $\sim 10^{13}$ Hz is due to dipolar relaxation (Debye relaxation)

PHASE-VELOCITY DISPERSION

- In imaging systems, phase-velocity dispersion causes light of different colors to be focused at different points
  - ▷ The refractive index n is a function of wavelength λ (or frequency f) because the phase velocity v depends on λ
  - ▷ One measure of phase-velocity dispersion is

$$\frac{dn}{d\omega} = \frac{n}{\beta}\left(\frac{1}{v_g} - \frac{1}{v}\right)$$

which contains both the group velocity

$$v_g = \left(\frac{d\beta}{d\omega}\right)^{-1} = \frac{d\omega}{d\beta}$$

and the phase velocity

$$v = \frac{\omega}{\beta}$$

SNELL'S LAWS

- The normal to a dielectric interface and the incident ray define the **plane of incidence**
  - ▷ The incident (i), reflected (r) and refracted (transmitted) (t) rays are all in the plane of incidence
  - ▷ Snell's law of reflection:

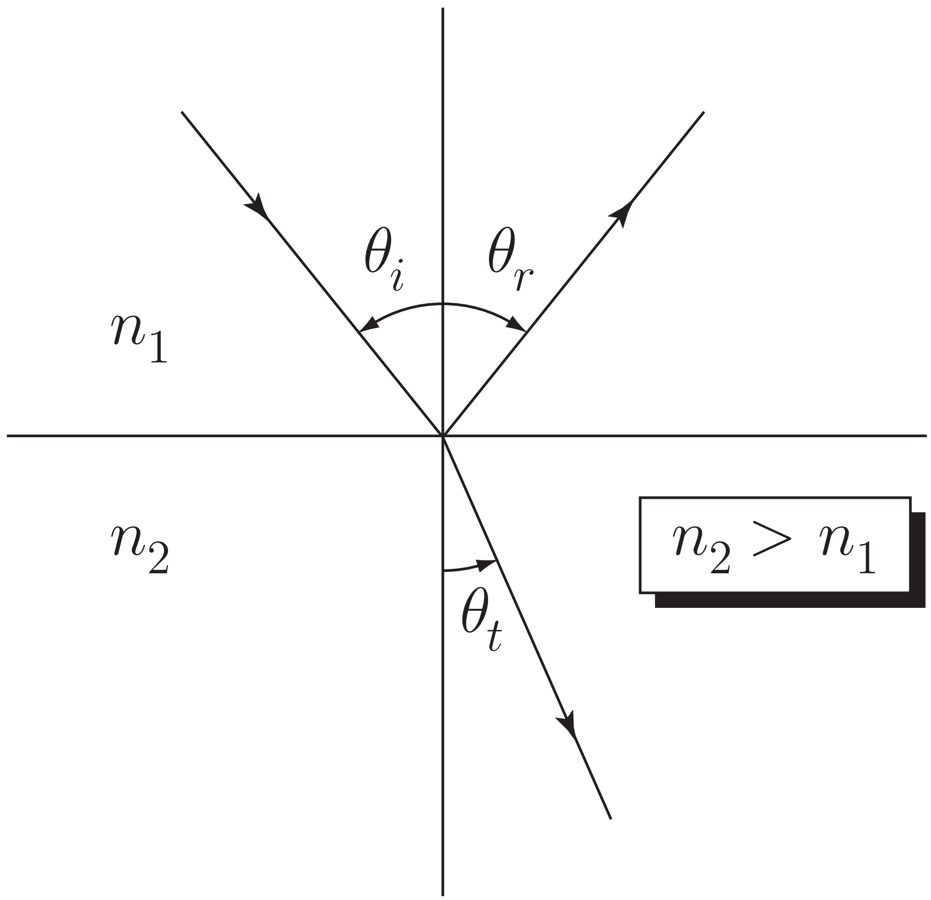
$$\theta_r = \theta_i$$

  - ▷ Snell's law of refraction:

$$n_2 \sin\theta_t = n_1 \sin\theta_i$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1} = \frac{v_1}{v_2}$$

ANGLES FOR SNELL'S LAWS



$n_2 > n_1$

TOTAL INTERNAL REFLECTION

- Occurs when a ray passes from a more dense medium into a less dense medium ($n_2 < n_1$)
  - ▷ From Snell's law of refraction,

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i > \sin\theta_i$$

  - ▷ When $\sin\theta_t = 1$, $\theta_t = 90^\circ \Rightarrow$ refracted ray goes along the interface
  - ▷ When this happens, θ_i is equal to the **critical angle** θ_c, where

$$\boxed{\sin\theta_c = \frac{n_2}{n_1}}$$

  - ▷ For rays such that $\theta_i > \theta_c$, there is no refracted ray
    - ◦ All energy is reflected $\Rightarrow$ **total internal reflection**
  - ▷ Main application: **Guided waves** in dielectric slabs and fibers

WHEN CAN ONE USE RAY OPTICS?

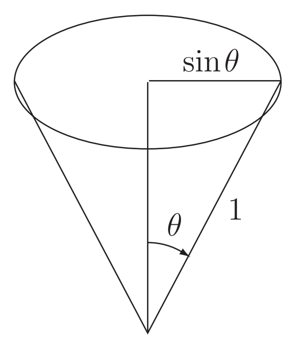
- One can use ray optics instead of wave optics:
  - ▷ Except near the edge of a shadow (get diffraction rings)
  - ▷ When λ is small compared to the sizes of all apertures
  - ▷ When interference and diffraction are negligible:
    - ∘ When $\lambda \ll w$ where $w =$ slit width, facet width, or grating period
  - ▷ At low bit rates
  - ▷ At the start of most lens designs, but perhaps not in the final stages

IMAGING ANALYSIS USING RAY OPTICS (1)

- Lenses are used for collimation and to focus light into fibers and waveguides
  - ▷ For a thin lens with spherical surfaces (radii of curvature R_1 and R_2), the focal length is
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
where n is the refractive index of the lens material
  - ▷ The distances d_o and d_i of an object and the image formed by a thin lens are given by the equation
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$
    - ◦ Linear magnification:
$$M = \frac{d_i}{d_o} = \frac{1}{(d_o/f) - 1}$$
    - ◦ Angular magnification:
$$\frac{1}{M} = \frac{\tan \alpha_i/2}{\tan \alpha_o/2}$$

IMAGING ANALYSIS USING RAY OPTICS (2)

- **Numerical aperture** of an optical system
  - ▷ Defined as
$$\text{NA} = n \sin\theta$$
where θ is the system's maximum acceptance half-angle and n is the refractive index of the outside material



  - ▷ If $n = 1$, then $0 \leq \text{NA} \leq 1$
    - ∘ 0 means that no light gets into the system
    - ∘ 1 means that all of the light propagating towards the system gets in
    - ∘ Power into system $\propto (\text{NA})^2$

WAVE OPTICS

- Experimental basis: Interference experiments of Young, diffraction and interference experiments of Fresnel
- Theoretical basis: Maxwell's equations
- Important wave phenomena in optics:
  - ▷ Polarization
  - ▷ Interference
  - ▷ Diffraction
  - ▷ Guided waves
    - ◦ Can also be understood semi-quantitatively using ray optics

MAXWELL'S EQUATIONS

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \cdot \mathbf{D} = \rho$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

- From the first line, the normal components of **D** and **B** are continuous across a dielectric interface
- From the second line, the tangential components of **E** and **H** are continuous across a dielectric interface

PLANE-WAVE SOLUTIONS OF MAXWELL'S EQUATIONS

- Assumptions:
  - ▷ Homogeneous, isotropic, lossless, dielectric medium
  - ▷ Plane wave propagating in the $+z$ direction
- Fields:

$$\mathbf{E} = \left(\hat{\mathbf{x}}E_1 + \hat{\mathbf{y}}E_2 e^{j\psi}\right) e^{j(\omega t - \beta z)} \qquad (E_1,\ E_2 \text{ real})$$

$$\mathbf{H} = \frac{1}{\eta}\left(-\hat{\mathbf{x}}E_2 e^{j\psi} + \hat{\mathbf{y}}E_1\right) e^{j(\omega t - \beta z)} = \frac{1}{\eta}\hat{\mathbf{z}} \times \mathbf{E}$$

  - ▷ The ratio of the magnitudes of $\mathbf{E}$ and $\mathbf{H}$ is the **characteristic impedance** of the medium,

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

  - ▷ Phase velocity:

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{\omega}{\beta}$$

  - ▷ Time-averaged Poynting vector:

$$\mathbf{S} = \frac{1}{2}\,\mathrm{Re}\left[\mathbf{E}^* \times \mathbf{H}\right] = \frac{1}{2}\left(\frac{E_1^2 + E_2^2}{\eta}\right)\hat{\mathbf{z}}$$

POLARIZATION IN COMMUNICATION SYSTEMS

- Many components and devices are polarization-sensitive
  - ▷ Ordinary (round) fiber scrambles the polarization of light passing through
  - ▷ Even a flat glass plate is polarization-sensitive!
    - ◦ Different polarizations have different reflectivities at oblique incidence
  - ▷ Different modes in an optical waveguide usually have different polarizations
    - ◦ Different polarizations have different group velocities in a fiber
      - ◇ Leads to polarization-mode dispersion and bandwidth limitations
    - ◦ Gain in a semiconductor optical amplifier is polarization-dependent
  - ▷ Raman gain is polarization-dependent

POLARIZATION (1)

- The **polarization** of a plane wave is determined by the value of the relative phase angle, ψ, and the ratio E_1/E_2, where

$$E_x = E_1 \cos(\omega t - \beta z), \qquad E_y = E_2 \cos(\omega t - \beta z + \psi)$$

- Plane polarization:

$$\psi = 0$$

  ▷ Real components of $\mathbf{E}$:

$$E_x = E_1 \cos(\omega t - \beta z), \qquad E_y = E_2 \cos(\omega t - \beta z)$$

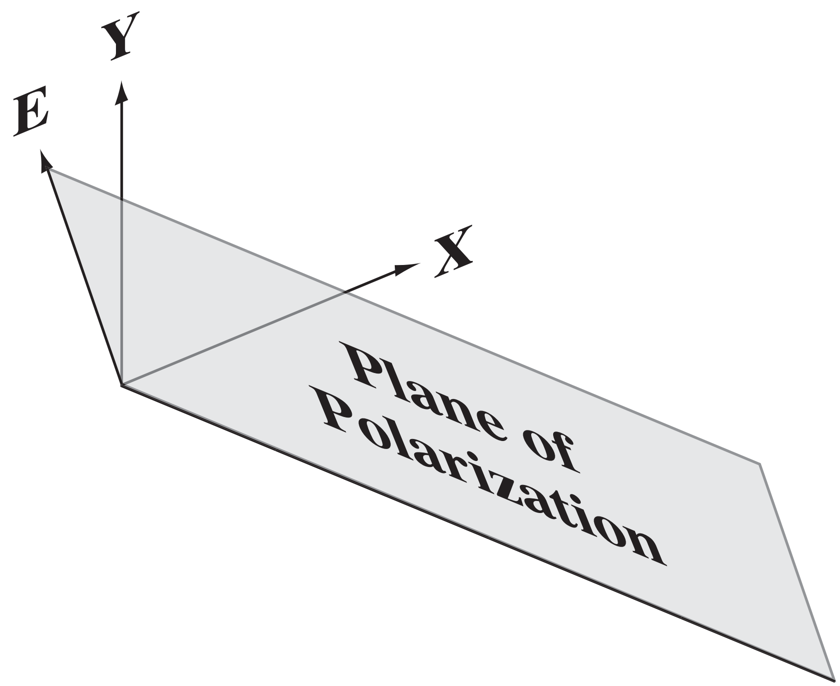
  ▷ The $\mathbf{E}$ vector makes an angle

$$\phi = \tan^{-1}(E_2/E_1)$$

  with the x axis

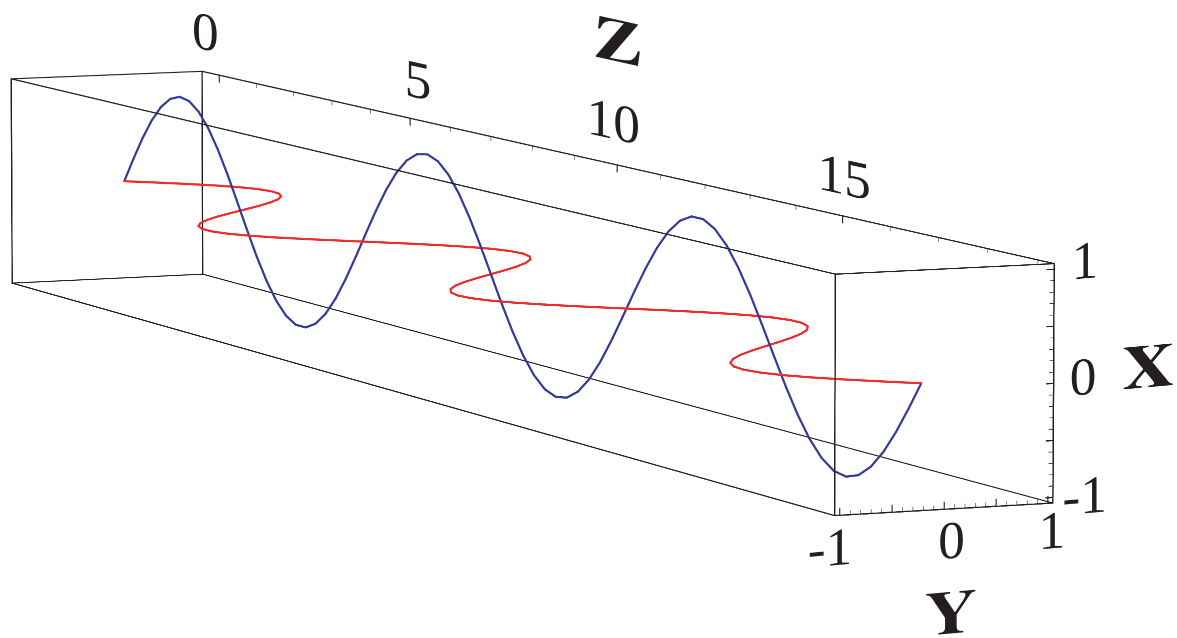
- Note that the word "plane" has been used with 2 different meanings on this slide!

PLANE OF POLARIZATION



- The direction of the electric field and the direction of propagation determine the **plane of polarization**

SNAPSHOT OF PLANE-POLARIZED PLANE WAVE



- Red curve: Tip of the **E** vector Blue curve: Tip of the **H** vector
  The plane of polarization is the $x - z$ plane

POLARIZATION (2)

- Circular polarization:

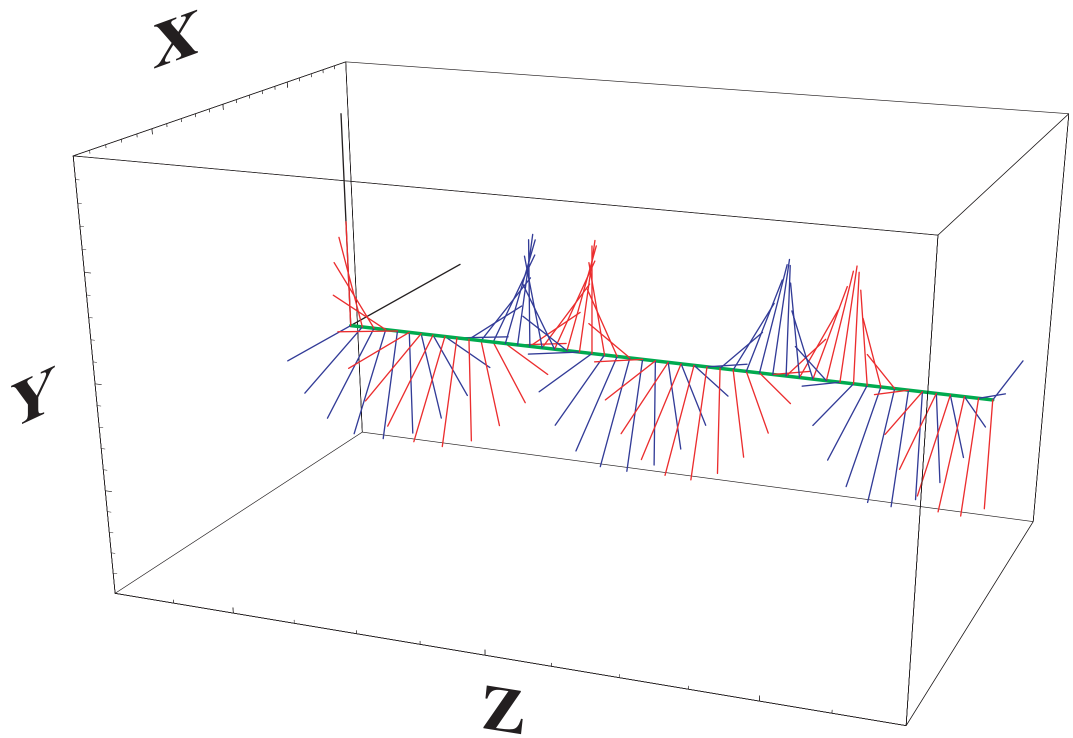
$$\psi = \pm\frac{\pi}{2}, \qquad E_2 = E_1$$

  - ▷ Real components of **E**:

$$E_x = E_1 \cos(\omega t - \beta z), \qquad E_y = \mp E_2 \sin(\omega t - \beta z)$$

  - ▷ For fixed z, the **E** vector describes a circle in the $x - y$ plane
    - ○ $\psi = \frac{\pi}{2}$: Right circular polarization
    - ○ $\psi = -\frac{\pi}{2}$: Left circular polarization
    - ○ The terms "right" and "left" circular polarization are confusing, because a right-circularly-polarized wave actually describes a left-handed screw as the wave propagates in z
      - ◇ For right-circularly-polarized light, the tip of the **E** vector rotates clockwise in the $x - y$ plane

SNAPSHOT OF CIRCULARLY-POLARIZED PLANE WAVE



- Red lines: **E** vectors   Blue lines: **H** vectors
  The light is left-circularly-polarized

POLARIZATION (3)

- Elliptical polarization:

$$E_2 \neq E_1, \quad \textbf{or} \quad E_2 = E_1 \quad \text{and} \quad \psi \neq \pm\frac{\pi}{2} \quad \text{and} \quad \psi \neq 0$$

▷ Real components of **E**:

$$E_x = E_1 \cos(\omega t - \beta z), \qquad E_y = E_2 \cos(\omega t - \beta z + \psi)$$

▷ For fixed z, the **E** vector describes an ellipse in the $x - y$ plane

∘ Eliminating $\zeta = \omega t - \beta z$ from the equations for E_x and E_y results in

$$\left(\frac{E_x}{E_1}\right)^2 + \left(\frac{E_y}{E_2}\right)^2 - 2\cos\psi \frac{E_x}{E_1}\frac{E_y}{E_2} = \sin^2\psi$$

which is the equation of an ellipse that makes an angle θ with the x axis, where

$$\tan 2\theta = \frac{2E_1E_2\cos\psi}{E_1^2 - E_2^2}$$

POLARIZATION (4)

- For a plane wave, there are always 2 orthogonal states of polarization
  - ▷ Simplest case: Orthogonal linear polarizations described by unit coordinate vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$
    - ◦ Orthogonality: $\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = 0$
    - ◦ Normalization: $\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = 1 = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}}$
    - ◦ Example of orthogonally polarized fields:

$$\mathbf{E}_1 = \hat{\mathbf{x}} E_1 e^{j(\omega t - \beta z)} \qquad \mathbf{E}_2 = \hat{\mathbf{y}} E_2 e^{j\psi} e^{j(\omega t - \beta z)}$$

  - ▷ The important point: **Waves with orthogonal polarizations do not interfere with one another**
- Example: In a "single-mode" fiber, there are really two modes, one for each of the two orthogonal polarizations
  - ▷ These modes don't have the same group velocity
  - ▷ The difference in group velocities leads to polarization-mode dispersion, pulse spreading, and bandwidth limitations

POLARIZATION (5)

- Orthogonal states of polarization in general
  - ▷ The polarization state of the elliptically polarized field

$$\mathbf{E} = \left(\hat{\mathbf{x}}E_1 + \hat{\mathbf{y}}E_2 e^{j\psi}\right) e^{j(\omega t - \beta z)} \qquad (E_1,\ E_2 \text{ real})$$

is described by the complex unit vector

$$\hat{\mathbf{e}}_1 = \frac{1}{\sqrt{E_1^2 + E_2^2}} \left(\hat{\mathbf{x}}E_1 + \hat{\mathbf{y}}E_2 e^{j\psi}\right)$$

  - ▷ The orthogonal polarization is described by the complex unit vector

$$\hat{\mathbf{e}}_2 = \hat{\mathbf{z}} \times \hat{\mathbf{e}}_1^* = \frac{1}{\sqrt{E_1^2 + E_2^2}} \left(-\hat{\mathbf{x}}E_2 e^{-j\psi} + \hat{\mathbf{y}}E_1\right)$$

    - ○ Orthogonality: $\hat{\mathbf{e}}_1^* \cdot \hat{\mathbf{e}}_2 = 0 = \hat{\mathbf{e}}_2^* \cdot \hat{\mathbf{e}}_1$
    - ○ Normalization: $\hat{\mathbf{e}}_1^* \cdot \hat{\mathbf{e}}_1 = 1 = \hat{\mathbf{e}}_2^* \cdot \hat{\mathbf{e}}_2$
    - ○ If $\psi \in (0, \pi)$, then $\hat{\mathbf{e}}_1$ describes a right elliptical polarization state, and $\hat{\mathbf{e}}_2$ describes a left elliptical polarization state

POLARIZATION (6)

- Example of orthogonal states of circular polarization
  - ▷ A right-circularly-polarized wave is described by the unit vector

$$\hat{\mathbf{e}}_R = \frac{1}{\sqrt{2}} (\hat{\mathbf{x}} + j\hat{\mathbf{y}})$$

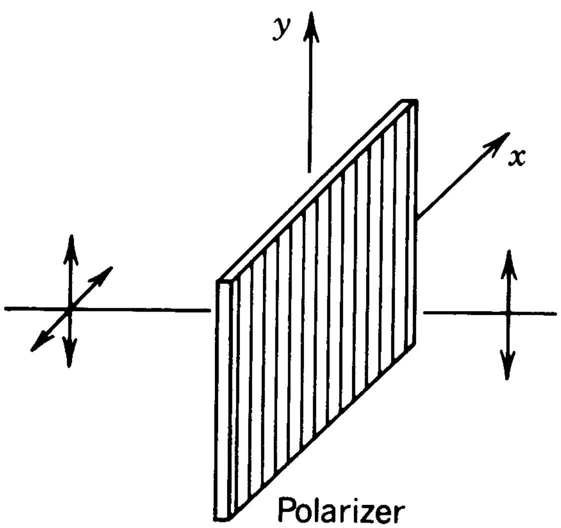
  - ▷ The orthogonal polarization is left-circularly-polarized and is described by the complex unit vector

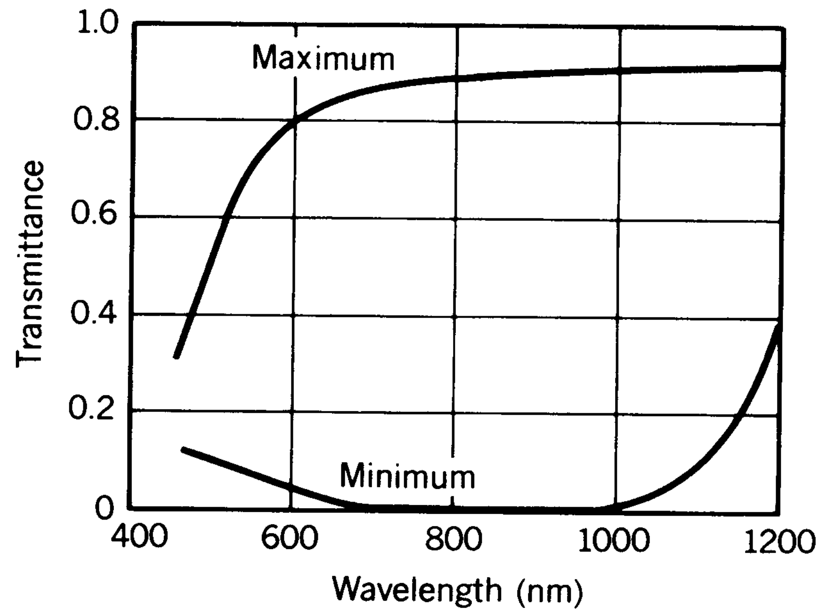
$$\hat{\mathbf{e}}_L = \frac{1}{\sqrt{2}} (j\hat{\mathbf{x}} + \hat{\mathbf{y}}) = \frac{j}{\sqrt{2}} (\hat{\mathbf{x}} - j\hat{\mathbf{y}})$$

POLARIZERS AND ANALYZERS

- A **polarizer** prepares a specific state of polarization
- An **analyzer** blocks a specific state of polarization and transmits the orthogonal polarization
- Materials and techniques for making polarizers and analyzers:
  - ▷ Polarization-selective absorption
    - ○ Some materials are **dichroic**
      - ◇ One of two states of polarization is absorbed more than the other
      - ◇ Example: Polaroid (as in sunglasses)
  - ▷ Brewster's-angle polarizers and analyzers
    - ○ Problem: Brewster's angle is wavelength-dependent
  - ▷ Polarizing prisms
    - ○ Best rejection ratio for the blocked polarization state
  - ▷ Wire-grid polarizers and analyzers
    - ○ Used in the far infrared (where there are few birefringent materials)

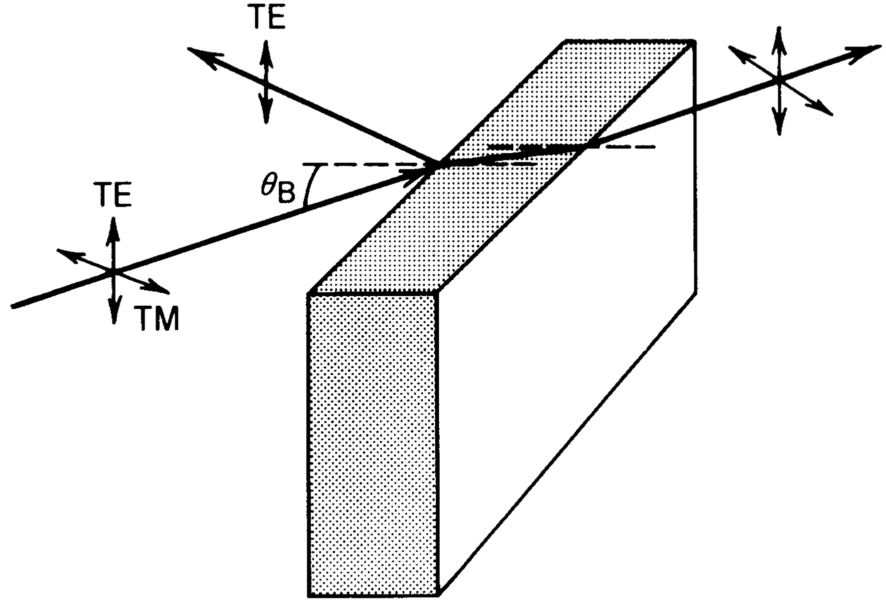
DICHROIC POLARIZERS





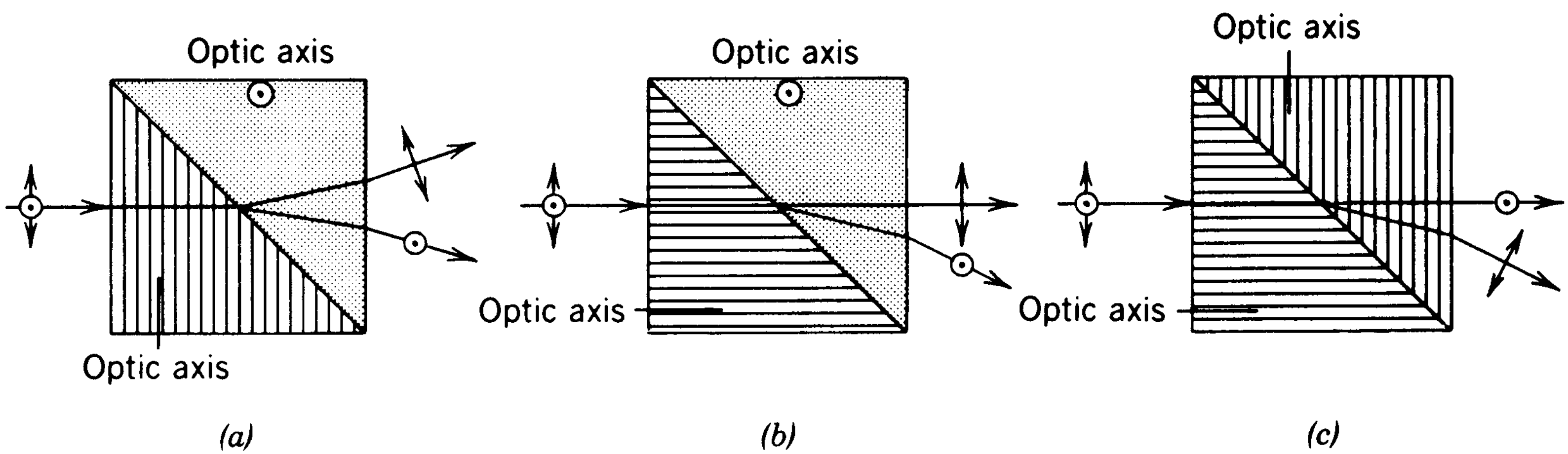
B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics* (Wiley, 1991), Fig. 6-6.1

BREWSTER'S-ANGLE POLARIZERS



B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics* (Wiley, 1991), Fig. 6-6.2

POLARIZING PRISMS



(a) Wollaston prism (b) Rochon prism (c) Sénarmont prism

B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics* (Wiley, 1991), Fig. 6-6.3

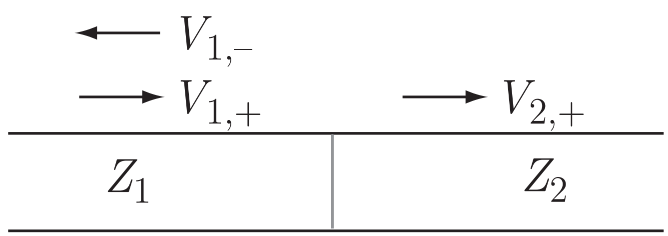
REFLECTION AT NORMAL INCIDENCE (1)

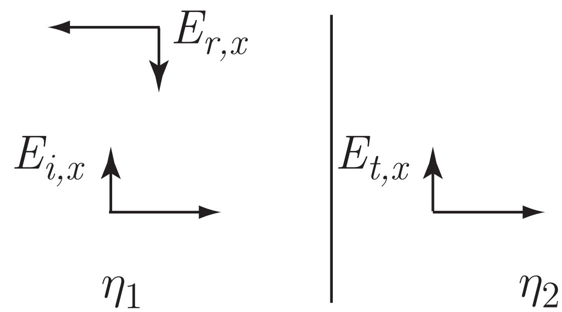
- Goal: Understand how to calculate the reflection coefficient at the interface between two dielectrics in terms of the indexes of refraction
- For normal incidence ($\theta_i = 0°$), the reflection coefficient of a plane wave is independent of the wave's polarization
  - ▷ A reflected wave propagates back into the medium in which the incident wave propagates
  - ▷ The **E** and **H** fields of the reflected and transmitted waves can be found by either of two methods:
    - ∘ Brute-force analysis of the boundary conditions (physics method)
    - ∘ Impedance analysis borrowed from transmission-line theory (EE method)

REFLECTION AT NORMAL INCIDENCE (2)

- Boundary conditions at a dielectric interface:
  - ▷ Continuity of the normal components of **D** and **B**
  - ▷ Continuity of the tangential components of **E** and **H**
- Fields:
  - ▷ In medium 1, $\mathbf{E} = \mathbf{E}_i + \mathbf{E}_r$
  - ▷ In medium 2, $\mathbf{E} = \mathbf{E}_t$
- For a plane wave at normal incidence ($\theta_i = 0°$), all of the fields are parallel (tangential) to the interface
  - ▷ The transmission-line approach is easy to justify and apply
  - ▷ The transmission-line equations are really just integral forms of Maxwell's equations
  - ▷ $E_{i,x}$ is analogous to $V_{1,+}$, $H_{i,y} = E_{i,x}/\eta$ is analogous to $I_{1,+} = V_{1,+}/Z_0$, η is analogous to Z_0, $E_{r,x}$ is analogous to $V_{1,-}$, etc.

REFLECTION AT NORMAL INCIDENCE (3)





- Boundary conditions:
  - ▷ V and I are continuous at the transmission-line interface
  - ▷ E_x and H_y are continuous at the dielectric interface
- Voltage reflection coefficient:

$$\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

- Electric-field reflection coefficient:

$$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{n_2} - \frac{1}{n_1}}{\frac{1}{n_2} + \frac{1}{n_1}} = -\frac{n_2 - n_1}{n_2 + n_1}$$

REFLECTION AT NORMAL INCIDENCE (4)

- Power (and intensity) reflection coefficient:

$$\rho^2 = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$

- Power (and intensity) transmission coefficient:

$$1 - \rho^2 = 1 - \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 = \frac{4n_1 n_2}{(n_2 + n_1)^2}$$

  ▷ Example: Glass-air interface ($n_1 = 1.0$, $n_2 = 1.5$)

$$\rho^2 = 0.04$$

REFLECTION AT OBLIQUE INCIDENCE (1)

- For all angles of incidence except 0° (normal incidence), the reflection coefficient of a plane wave depends on the wave's polarization
  - ▷ The useful orthogonal states of polarization for this situation are called TE and TM
    - ○ In TE (or $\perp$) polarization, the **E** field is perpendicular to the plane of incidence
      - ◇ The plane of incidence is the plane defined by the direction of the incident field and the normal to the dielectric surface
    - ○ In TM (or $\|$) polarization, the **E** field is in the plane of incidence

REFLECTION AT OBLIQUE INCIDENCE (2)

- Impedance analysis
  - ▷ Apply to the tangential components of **E** and **H**
    - ∘ The tangential components are continuous at a dielectric interface, like V and I in a transmission line
  - ▷ In the medium from which the wave is incident,

$$Z_{1,\parallel} = \frac{E_{i,x}}{H_{i,x}} = -\frac{E_{r,x}}{H_{r,x}} = \eta_1 \cos\theta$$

$$Z_{2,\perp} = -\frac{E_{i,y}}{H_{i,y}} = \frac{E_{r,y}}{H_{r,y}} = \eta_1 \sec\theta$$

(the plane of incidence is the $x - z$ plane)

  - ▷ Reflection coefficient for TM wave (for example):

$$\rho_\parallel = \frac{Z_{2,\parallel} - Z_{1,\parallel}}{Z_{2,\parallel} + Z_{1,\parallel}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

FRESNEL FORMULAS

- The normal to a dielectric interface and the incident ray define the **plane of incidence**
  - ▷ $\mathbf{E}$ components that are in (parallel to) the plane of incidence do not interfere with field components that are perpendicular to the plane of incidence
  - ▷ Transmission and reflection coefficients:

$$\tau_\parallel = \frac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} = \frac{2 \sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

$$\tau_\perp = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{2 \sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)}$$

$$\rho_\parallel = -\frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} = -\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$\rho_\perp = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

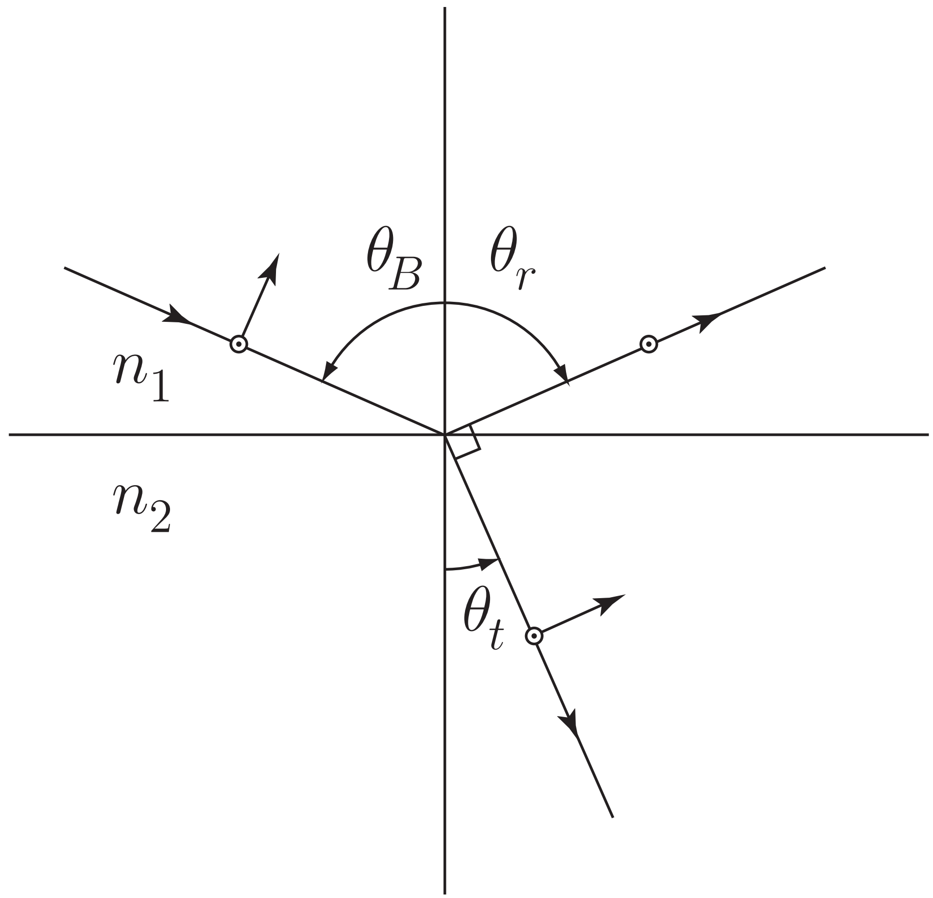
BREWSTER'S ANGLE (1)

- When $\theta_i + \theta_t = \pi/2$, the **E** component in the plane of incidence is not reflected ($\rho_\parallel = 0$)
  - ▷ A more usable formula for Brewster's angle:

$$\tan\theta_B = \frac{n_t}{n_i}$$

  - ▷ At Brewster's angle, the reflected light is fully polarized perpendicular to the plane of incidence
    - Polaroid sunglasses and camera filters
    - End faces on lasers

BREWSTER'S ANGLE (2)



OPTICAL POWER

- The optical power per unit area incident on a surface is

$$|\mathbf{S} \cdot \hat{\mathbf{n}}| = |\hat{\mathbf{n}} \cdot (\mathbf{E} \times \mathbf{H})|$$

where $\mathbf{S}$ is the Poynting vector and $\hat{\mathbf{n}}$ is the unit normal to the surface

  ▷ For a plane wave propagating in the $+z$ direction,

$$\mathbf{S} = \frac{1}{\eta}\mathbf{E} \times (\hat{\mathbf{z}} \times \mathbf{E}) = \frac{1}{\eta}(\mathbf{E} \cdot \mathbf{E})\hat{\mathbf{z}}$$

  since $\hat{\mathbf{z}} \cdot \mathbf{E} = 0$

  ▷ The (optical) **intensity** is defined as

$$I = |\mathbf{S}| \qquad \text{(units are W/m}^2\text{)}$$

    ◦ For a plane wave,

$$I = \frac{1}{\eta}(\mathbf{E} \cdot \mathbf{E})$$

INTERFERENCE (1)

- **Interference** is the name given to the following optical phenomenon:
  If light from a source is split into two beams of intensities I_1 and I_2, and the beams are then recombined, the optical intensity in the superposition region varies from a maximum that is greater than $I_1 + I_2$ to a minimum that may be zero.
  - ▷ Electric field in superposition region:
    $$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$
    - ○ $\mathbf{E}_1$ and $\mathbf{E}_2$ may interfere constructively or destructively
      - ◇ The interference is manifested by differences between the total intensity and the sum of the intensities of the two beams
    - ○ Total intensity (not time-averaged, hence no factor of $\frac{1}{2}$):
      $$I = \frac{1}{\eta}\mathbf{E} \cdot \mathbf{E} = \frac{1}{\eta}(\mathbf{E}_1 \cdot \mathbf{E}_1 + \mathbf{E}_2 \cdot \mathbf{E}_2 + 2\mathbf{E}_1 \cdot \mathbf{E}_2)$$
    - ○ Intensity after time averaging over a few optical cycles:
      $$\overline{I} = \overline{I}_1 + \overline{I}_2 + \frac{2}{\eta}\overline{\mathbf{E}_1 \cdot \mathbf{E}_2}$$

INTERFERENCE (2)

- Intensity after time averaging over a few optical cycles:

$$\overline{I} = \overline{I}_1 + \overline{I}_2 + J_{12}$$

▷ $J_{12} = (2/\eta)\overline{\mathbf{E}_1 \cdot \mathbf{E}_2}$ is the **interference term**

▷ For plane waves propagating in the $+z$ direction:

$$\mathbf{E}_1 = \hat{\mathbf{e}}_1 E_1 \cos(\omega t - \beta z) \qquad \mathbf{E}_2 = \hat{\mathbf{e}}_2 E_2 \cos(\omega t - \beta z + \delta)$$

where δ is the **phase difference** between the two beams

▷ Resulting time average:

$$\begin{aligned}\overline{\mathbf{E}_1 \cdot \mathbf{E}_2} &= (\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_2) E_1 E_2 \overline{\cos(\omega t - \beta z)[\cos(\omega t - \beta z)\cos\delta - \sin(\omega t - \beta z)\sin\delta]} \\ &= \frac{1}{2}(\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_2) E_1 E_2 \cos\delta\end{aligned}$$

▷ Interference term:

$$J_{12} = \frac{1}{\eta}(\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_2) E_1 E_2 \cos\delta$$

INTERFERENCE (3)

- Interference of two plane waves:

$$\overline{I} = \frac{1}{2\eta}(E_1^2 + E_2^2) + J_{12}$$

  - ▷ Interference term:

$$J_{12} = \frac{1}{\eta}(\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_2) E_1 E_2 \cos\delta$$

  - ▷ **Orthogonally polarized waves** $(\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_2 = 0)$ **do not interfere**
  - ▷ The interference term takes on its maximum value when the waves are in phase, $\delta = 0 (\Rightarrow \cos\delta = 1)$, and its minimum value when the waves are out of phase, $\delta = \pi (\Rightarrow \cos\delta = -1)$

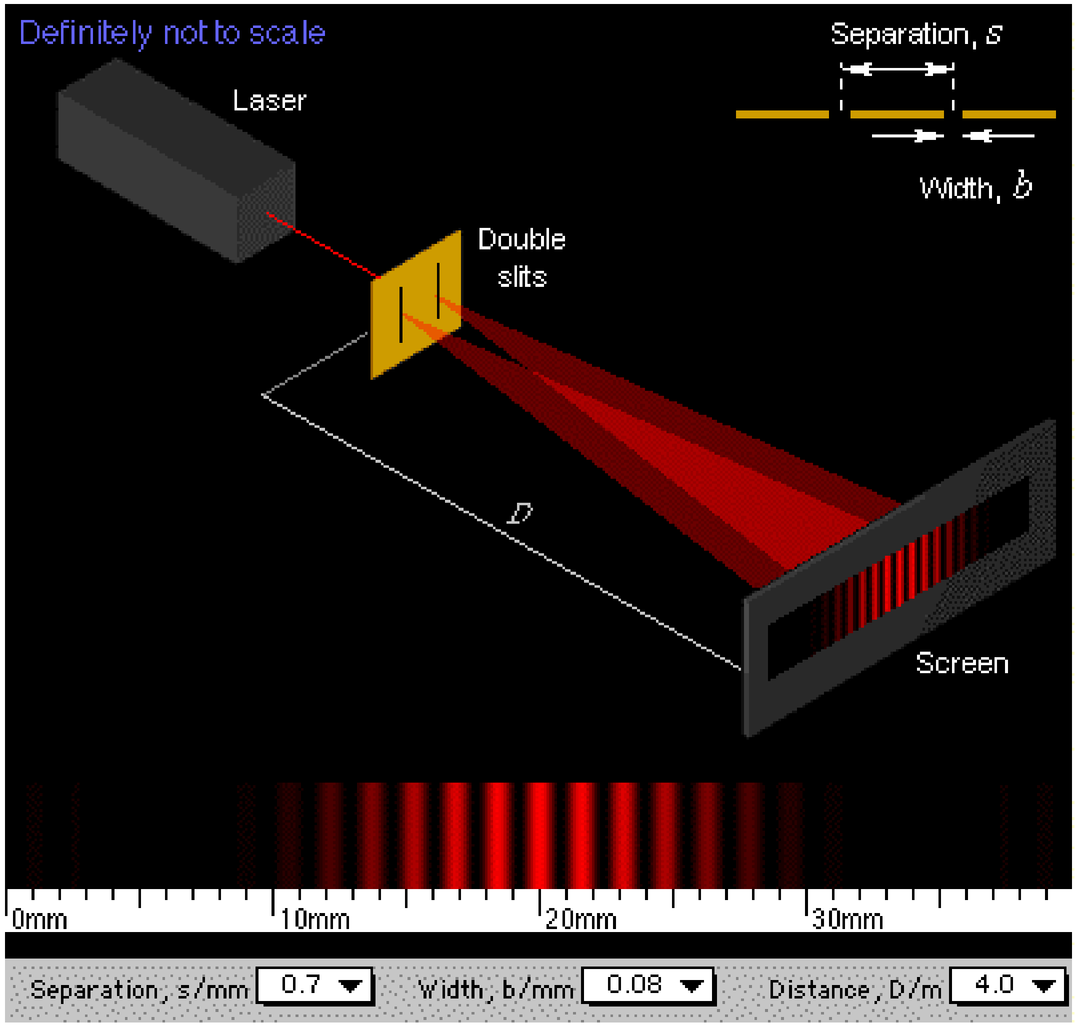
$$\overline{I}_{\text{max}} = \frac{1}{2\eta}(E_1 + E_2)^2$$

$$\overline{I}_{\text{min}} = \frac{1}{2\eta}(E_1 - E_2)^2$$

INTERFEROMETERS (1)

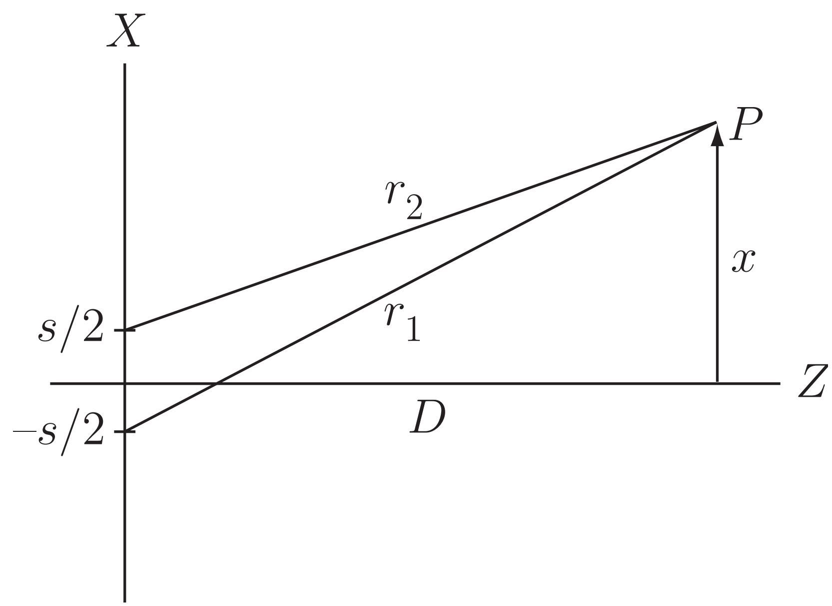
- Wavefront-splitting
  - ▷ Young's 2-slit interference experiment
- Amplitude-splitting
  - ▷ Michelson interferometer
  - ▷ Mach-Zehnder interferometer
  - ▷ Fabry-Pérot interferometer

YOUNG'S 2-SLIT INTERFERENCE EXPERIMENT (1)



http://www.matter.org.uk/schools/Content/Interference/laserinterference.html

YOUNG'S 2-SLIT INTERFERENCE EXPERIMENT (2)



$$r_1 = \sqrt{\left(x + \frac{s}{2}\right)^2 + D^2 + y^2}, \qquad r_2 = \sqrt{\left(x - \frac{s}{2}\right)^2 + D^2 + y^2}$$

$$\Rightarrow r_1^2 - r_2^2 = 2sx$$

$$\text{But } r_1^2 - r_2^2 = (r_1 - r_2)(r_1 + r_2) \approx (r_1 - r_2)2D$$

$$\Rightarrow r_1 - r_2 \approx \frac{sx}{D}$$

YOUNG'S 2-SLIT INTERFERENCE EXPERIMENT (3)

- The difference of distances from the two slits to the observation point P is

$$r_1 - r_2 \approx \frac{sx}{D}$$

where

$$x = \text{position on screen}$$
$$s = \text{spacing between slits}$$
$$D = \text{distance from slits to observation screen}$$

- Phase difference between waves arriving at P from the two slits:

$$\delta = \frac{2\pi n}{\lambda_0}(r_1 - r_2) \approx \frac{2\pi n s}{\lambda_0 D} x = \kappa x$$

  - ▷ This implies intereference fringes that are equally spaced in x
  - ▷ Use general formula for $\overline{I}$, setting $E_1 = E_2$ and $\hat{\mathbf{e}}_1 = \hat{\mathbf{e}}_2$ since waves from two slits have same amplitude and polarization
  - ▷ Result: Intensity $\overline{I} \propto 1 + \cos(\kappa x)$

INTERFERENCE OF LIGHT FROM N SLITS

- Let δ be the phase difference between waves arriving from adjacent slits

$$\delta = \frac{2\pi}{\lambda} s \sin\theta$$

  where θ is the angle that the waves make with the normal to the screen

- Intensity at P is

$$I(P) \propto \left| e^{j\delta} + e^{2j\delta} + \cdots + e^{(N-1)j\delta} \right|^2 = \left(\frac{\sin(N\delta/2)}{\sin \frac{1}{2}\delta} \right)^2$$

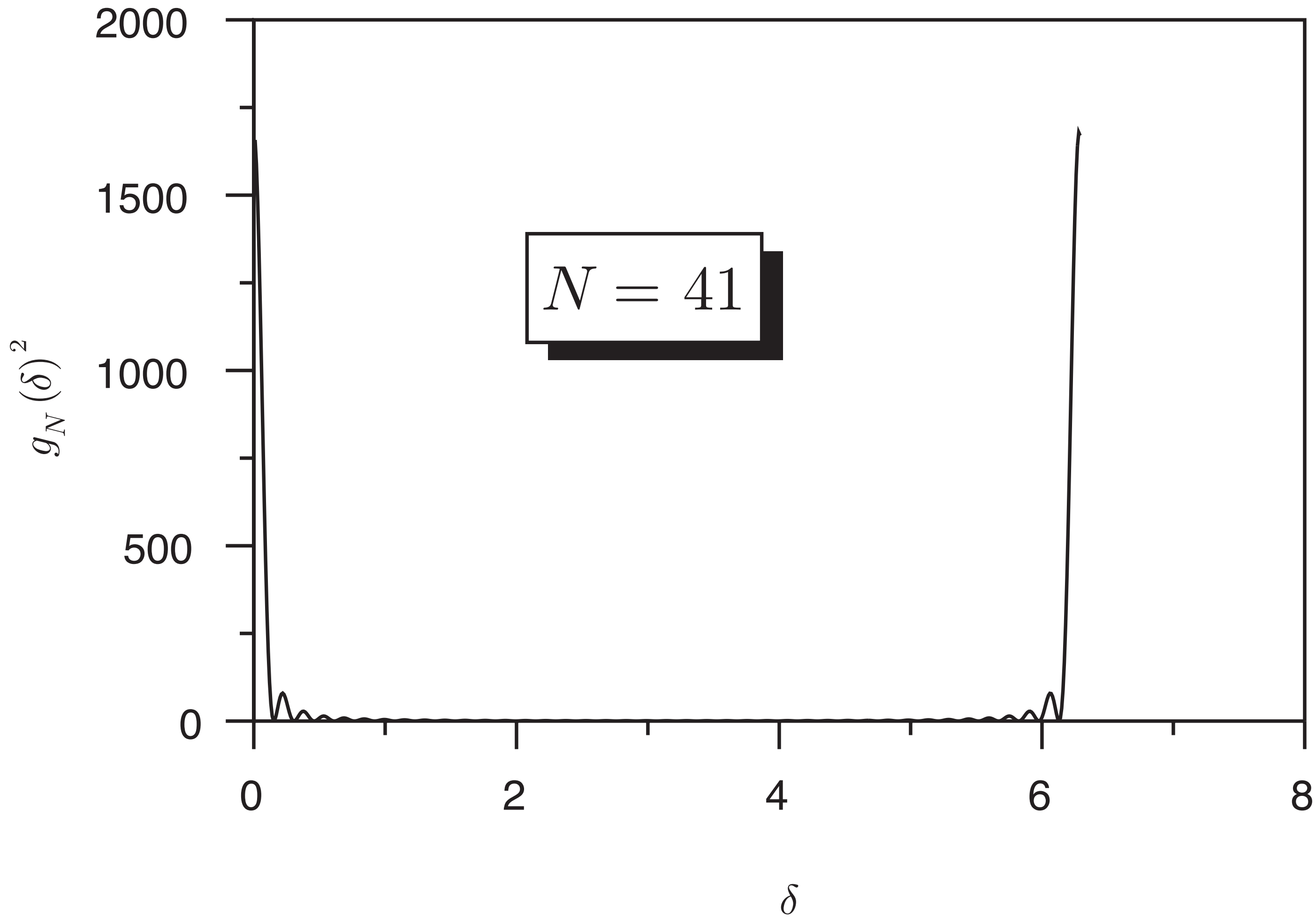
  ▷ This implies very sharp maxima when $\delta = 2m\pi$, *i.e.*, when

$$s \sin\theta = m\lambda$$

  ▷ The same analysis applies to simple **diffraction gratings**

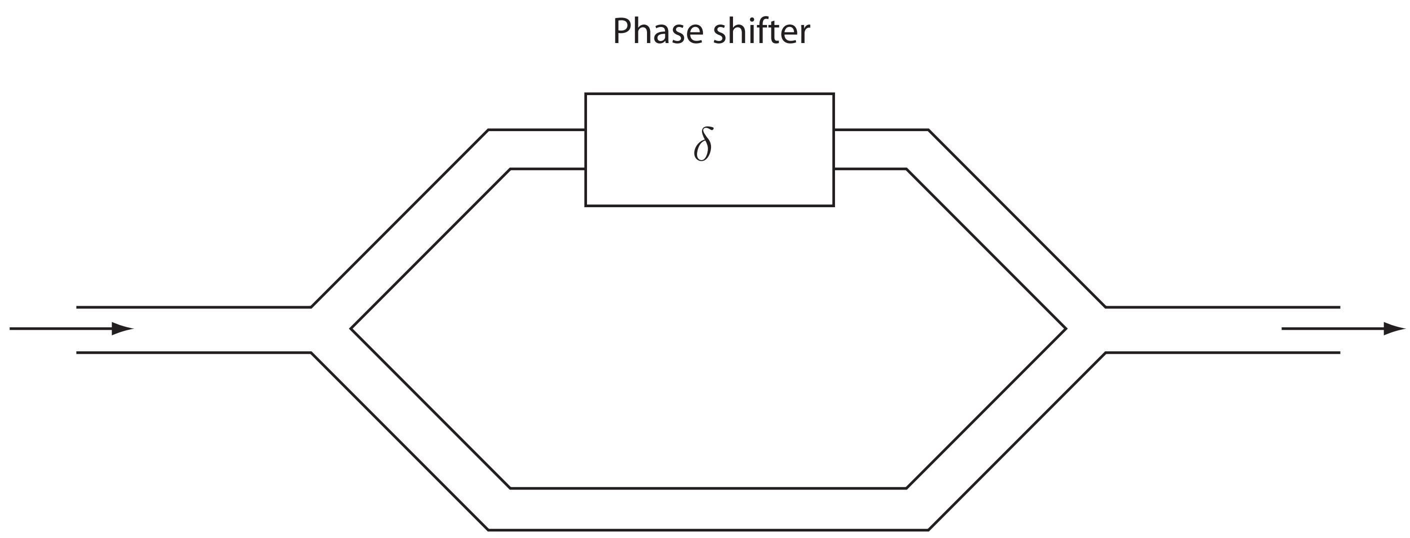
GRATING FUNCTION

$$g_N(\delta)^2 = \left(\frac{\sin\,(N\delta/2\,)}{\sin(\frac{1}{2}\delta)}\right)^2$$



INTERFEROMETERS (2)

- Optical waveguide Mach-Zehnder interferometer
  - ▷ A phase shifter (usually electro-optic) is incorporated into one of the arms of the interferometer
  - ▷ The output of the interferometer is a maximum when the phase difference δ between the beams arriving from the two arms is an integral multiple of 2π



INTERFEROMETERS (3)

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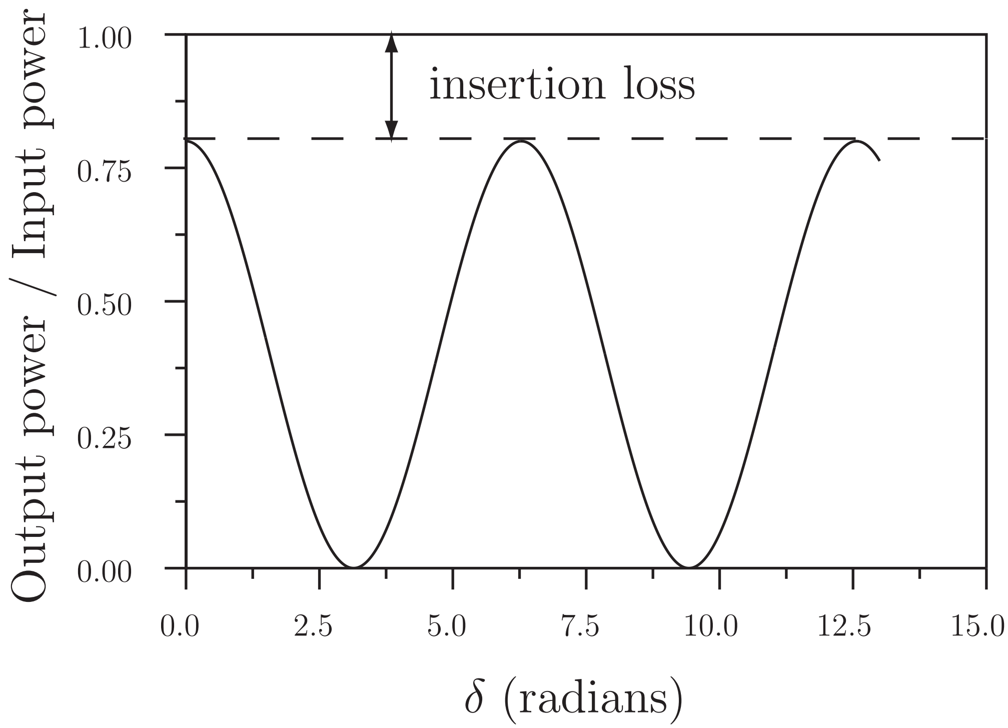
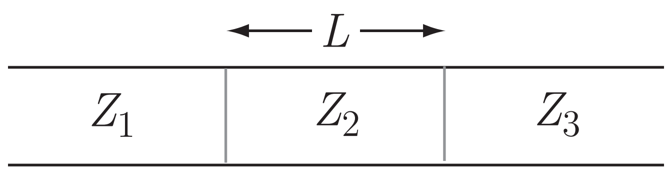
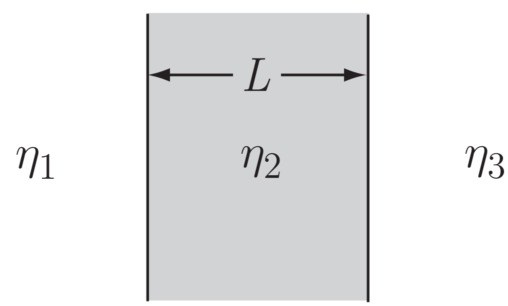


INTERFEROMETERS (4)

- The **Fabry-Pérot interferometer** is formed by two parallel, flat reflecting surfaces
  - ▷ The reflectivity at one or both surfaces may be just the Fresnel reflectivity at a dielectric interface, or may be enhanced with coatings
  - ▷ The transmission and reflection coefficients of the interferometer can be found by either of two methods:
    - ◦ Brute-force summation of multiple reflections (physics method)
    - ◦ Impedance analysis borrowed from transmission-line theory (EE method)
  - ▷ Application: **Longitudinal modes in semiconductor lasers**





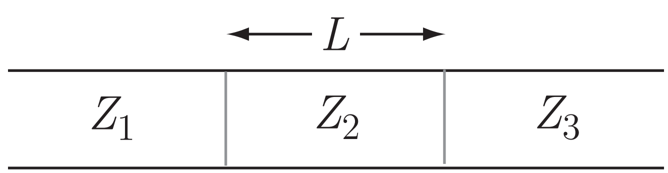
INTERFEROMETERS (5)

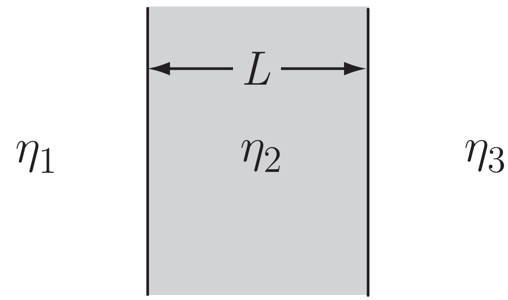
- Impedance analysis of the Fabry-Pérot interferometer
  - ▷ Use formula for the reflection coefficient for waves traveling from section 1 to section 2 for sinusoidal (single-frequency) waves on transmission lines:

$$\rho = \frac{Z_L - \eta_1}{Z_L + \eta_1}$$

where the complex input impedance at the end of section 1 is

$$Z_L = \eta_2 \left(\frac{\eta_3 \cos(k_2 L) + j\eta_2 \sin(k_2 L)}{\eta_2 \cos(k_2 L) + j\eta_3 \sin(k_2 L)} \right)$$





INTERFEROMETERS (6)

- Reflection coefficient at the entrance face of a dielectric layer (section 2) positioned between sections 1 and 3:

$$\rho = \frac{Z_L - \eta_1}{Z_L + \eta_1} = \frac{\eta_2(\eta_3 - \eta_1)\cos(k_2 L) + j(\eta_2^2 - \eta_1\eta_3)\sin(k_2 L)}{\eta_2(\eta_3 + \eta_1)\cos(k_2 L) + j(\eta_2^2 + \eta_1\eta_3)\sin(k_2 L)}$$

There are 2 important special cases for **normal incidence**:

▷ Anti-reflection coating

- The optical equivalent of a quarter-wavelength matching section in a transmission line

$$k_2 L = \frac{\pi}{2} \Rightarrow L = \frac{\lambda}{4}$$

- Fabry-Pérot interferometer

$$\eta_3 = \eta_1$$

(same media in the entrance and exit sections)

ANTI-REFLECTION COATING

- Substitute
$$k_2 L = \frac{\pi}{2} \Rightarrow \cos(k_2 L) = 0$$
into the formula for the reflection coefficient,
$$\rho = \frac{Z_L - \eta_1}{Z_L + \eta_1} = \frac{\eta_2(\eta_3 - \eta_1)\cos(k_2 L) + j(\eta_2^2 - \eta_1\eta_3)\sin(k_2 L)}{\eta_2(\eta_3 + \eta_1)\cos(k_2 L) + j(\eta_2^2 + \eta_1\eta_3)\sin(k_2 L)},$$
to get
$$\rho = \frac{\eta_2^2 - \eta_1\eta_3}{\eta_2^2 + \eta_1\eta_3} = \frac{n_1 n_3 - n_2^2}{n_1 n_3 + n_2^2}$$
  - ▷ The reflection coefficient is zero if $\eta_2^2 = \eta_1\eta_3$, *i.e.* if
$$n_2 = \sqrt{n_1 n_3}$$
    - ◦ This works only at one wavelength because $L = \frac{1}{4}\lambda$
    - ◦ It may not be possible to find a material such that $n_2 = \sqrt{n_1 n_3}$
    - ◦ Example: The power (and intensity) reflection coefficient of a quarter-wave MgF_2 coating ($n_2 = 1.38$) on glass ($n_3 \approx 1.50$, $\sqrt{1.50} \approx 1.22$) is $\rho^2 = 0.01$, versus $\rho^2 = 0.04$ for uncoated glass in air ($n_1 = 1.00$)

INTERFEROMETERS (7)

- Fabry-Pérot interferometer
  - ▷ Entrance and exit media are the same ($n_3 = n_1$)
  - ▷ Electric-field reflection coefficient in the entrance medium:

$$\rho = j\frac{(\eta_2^2 - \eta_1^2)\sin(k_2 L)}{2\eta_1\eta_2\cos(k_2 L) + j(\eta_2^2 + \eta_1^2)\sin(k_2 L)}$$

  - ▷ Power (and intensity) reflection coefficient in the entrance medium:

$$|\rho|^2 = \frac{(\eta_2^2 - \eta_1^2)^2\sin^2(k_2 L)}{4\eta_1^2\eta_2^2 + (\eta_2^2 - \eta_1^2)^2\sin^2(k_2 L)} = \frac{(\rho_{12}\rho_{23})^2\sin^2(k_2 L)}{\frac{1}{4}(1 + \rho_{12}\rho_{23})^2 + (\rho_{12}\rho_{23})^2\sin^2(k_2 L)}$$

  - ▷ Power (and intensity) transmission coefficient into the exit medium:

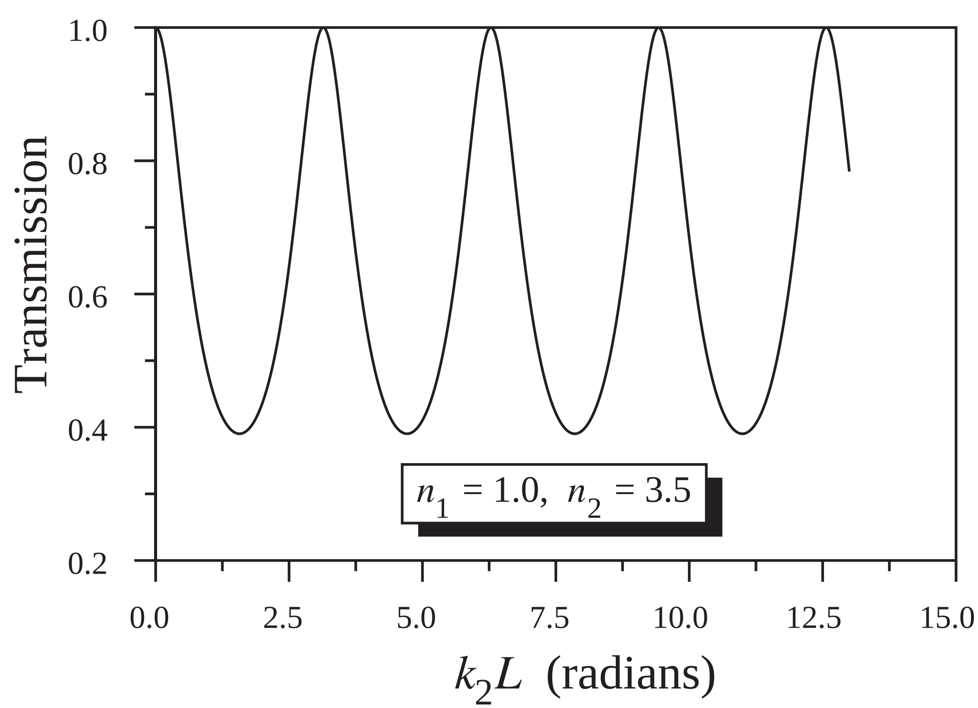
$$1 - |\rho|^2 = \frac{4\eta_1^2\eta_2^2}{4\eta_1^2\eta_2^2 + (\eta_2^2 - \eta_1^2)^2\sin^2(k_2 L)} = \frac{\frac{1}{4}(1 + \rho_{12}\rho_{23})^2}{\frac{1}{4}(1 + \rho_{12}\rho_{23})^2 + (\rho_{12}\rho_{23})^2\sin^2(k_2 L)}$$

where

$$\rho_{12} = \frac{n_1 - n_2}{n_1 + n_2}, \qquad \rho_{23} = -\rho_{12}$$

INTERFEROMETERS (8)

- Fabry-Pérot interferometer
  - ▷ The transmission of the interferometer is a maximum when the phase change k_2L of a beam traversing the plate is an integral multiple of π
  - ▷ This example applies to GaAs and related compound semiconductors



LONGITUDINAL MODES

- Fabry-Pérot interferometer
  - ▷ Transmission is a maximum when

$$k_2 L = m\pi \Rightarrow \frac{2n_2\pi}{\lambda} L = m\pi \Rightarrow 2L = m\frac{\lambda}{n_2}$$

    i.e., when **the round-trip distance in the interferometer is an integral multiple of the wavelength in the medium**
  - ▷ This is the condition for constructive interference of a wave that has just entered the interferometer with a wave that has traversed the interferometer once
  - ▷ The mth longitudinal mode has a wavelength λ_m and a frequency f_m

$$\lambda_m = \frac{2n_2 L}{m} \Rightarrow f_m = m\frac{c}{2n_2 L}$$

  - ▷ **Frequency spacing between adjacent longitudinal modes**:

$$\Delta f = \frac{c}{2n_2 L}$$

DIFFRACTION (1)

- **Diffraction** is the name given to the deviation of light from rectilinear propagation
  - ▷ A property of all waves
  - ▷ Usually most evident at the edges of shadows
  - ▷ Vital for a quantitative understanding of free-space optical beam propagation
    - ∘ Light exiting into air from a waveguide diffracts strongly
- Diffractive effects are already included in guided modes, so don't need to treat diffraction separately for light inside a waveguide
- Important special cases
  - ▷ Diffraction by a circular aperture — Airy disk
  - ▷ Diffraction by a rectangular aperture
  - ▷ Optics of Gaussian beams

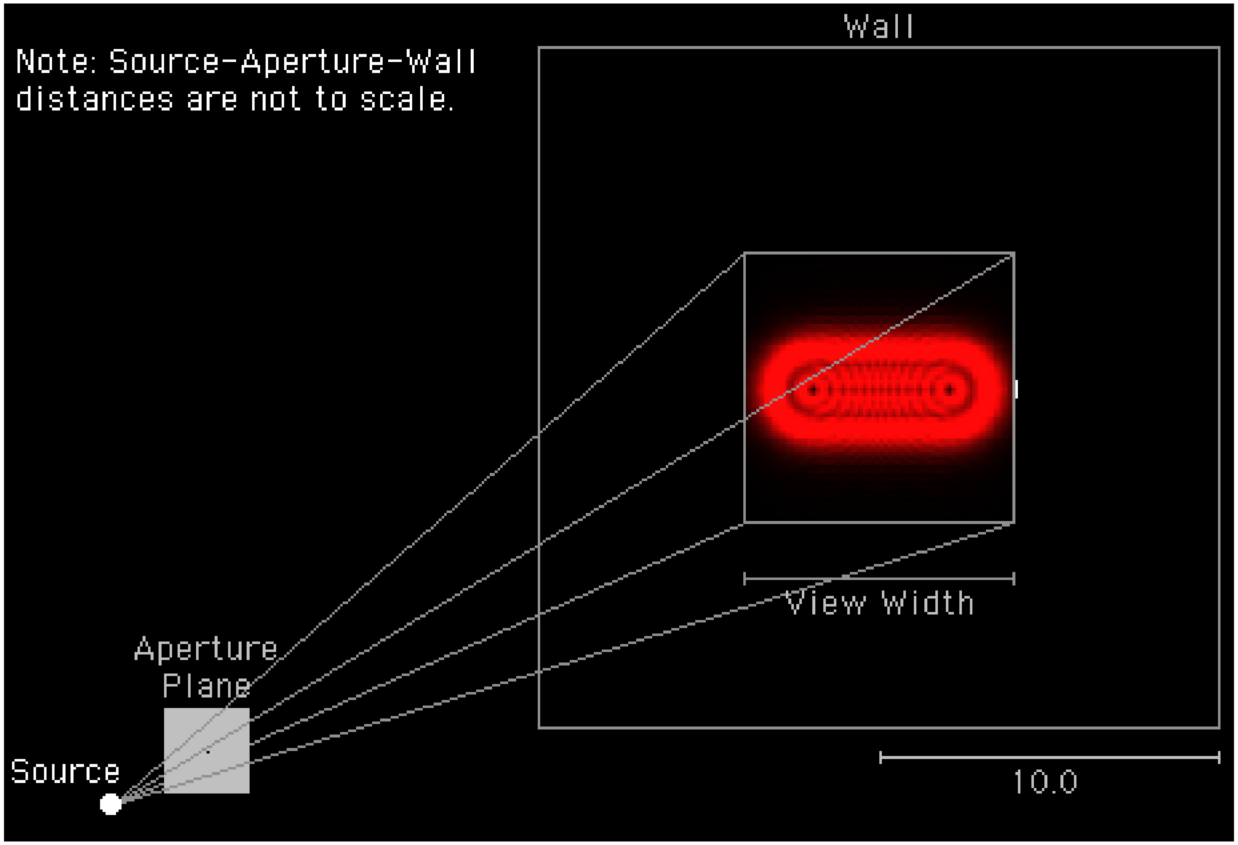
DIFFRACTION (2)

- The details of diffraction patterns depend on the **Fresnel number**

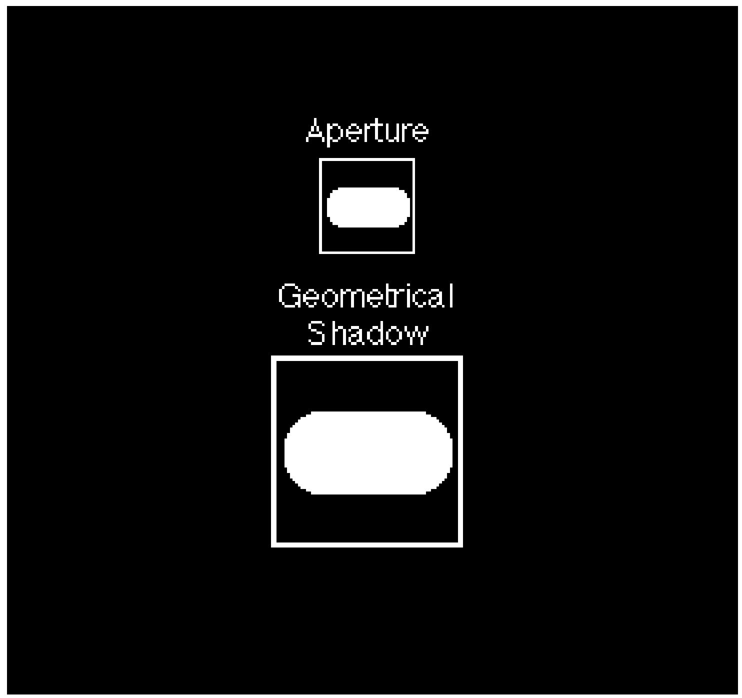
$$N = \frac{a^2}{\lambda d}$$

  - ▷ a is the radius of the aperture (we're assuming a circular aperture here...)
  - ▷ d is the distance to the observation point, or the focal length of a lens, if one is used; λ is the wavelength
  - ▷ Large Fresnel numbers ($N \gg 1$): **Fresnel regime**
    - ∘ Wavefront curvature is an essential part of the physics
    - ∘ Crudely, Fresnel diffraction by an aperture or obstacle results in a shadow with lots of bright and dark fringes
  - ▷ Small Fresnel numbers ($N \ll 1$): **Fraunhofer regime**
    - ∘ Plane-wave limit
    - ∘ The diffraction pattern is the Fourier transform of the aperture
    - ∘ The focal plane of an ideal lens is in the Frauhofer regime — a lens is a Fourier transformer (see Jack Gaskill's book)

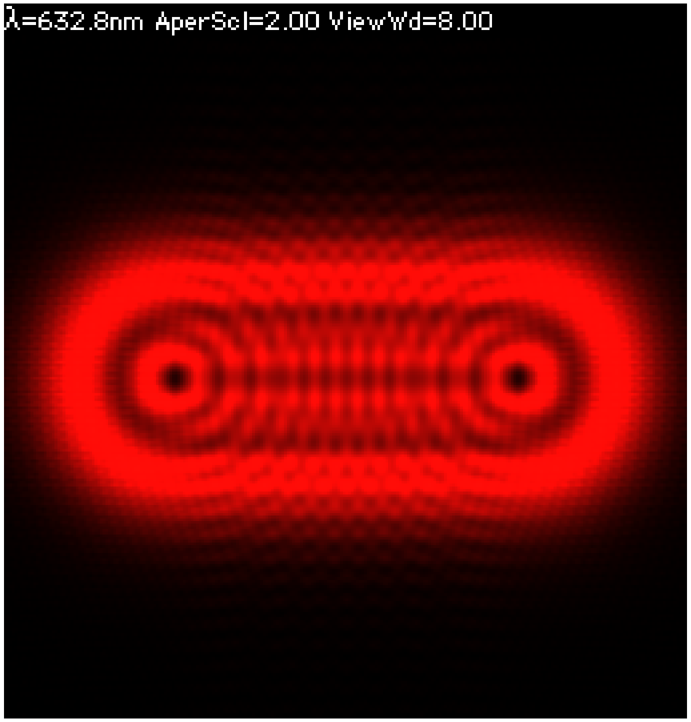
FRESNEL DIFFRACTION EXPERIMENT



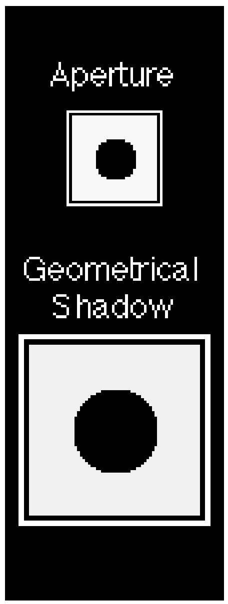
OBLONG APERTURE AND GEOMETRICAL SHADOW



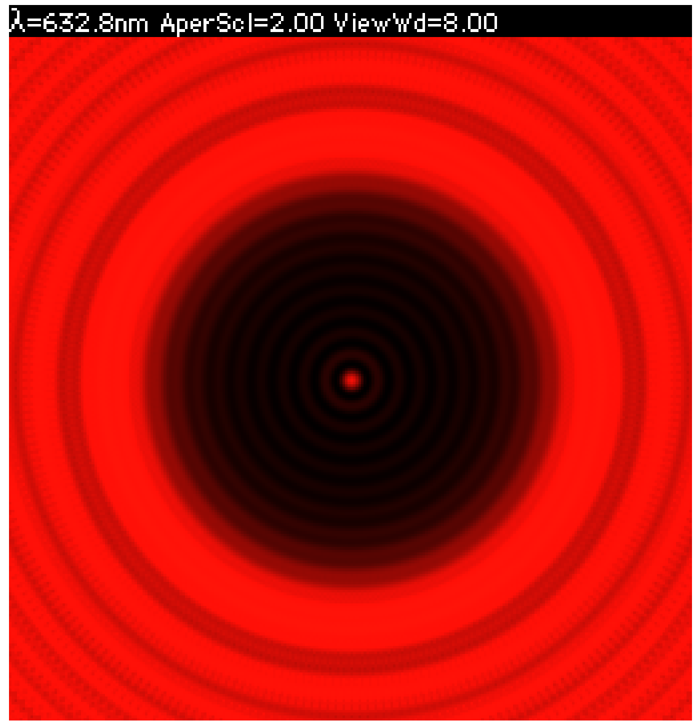
FRESNEL DIFFRACTION BY AN OBLONG APERTURE



FRESNEL DIFFRACTION BY A CIRCULAR OBSTACLE

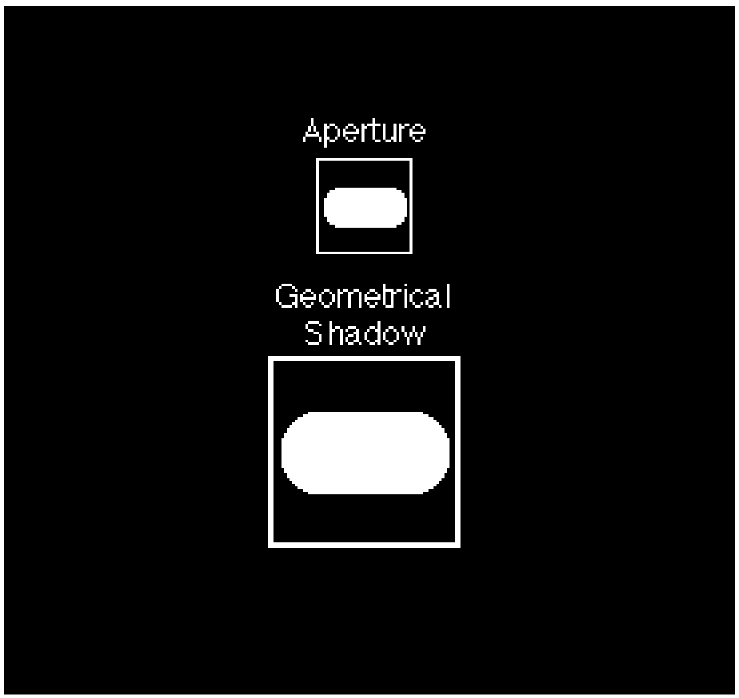


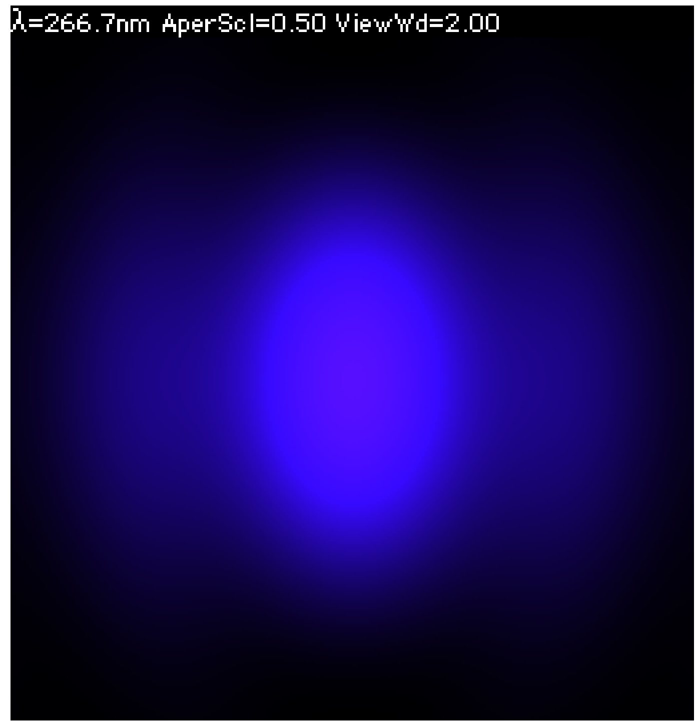
Fresnel diffraction pattern



Notice the "Poisson bright spot" in the center of the geometrical shadow!

FRAUNHOFER DIFFRACTION BY AN OBLONG APERTURE





Note that the diffraction pattern is widest in the direction in which the aperture is narrowest

FRAUNHOFER DIFFRACTION

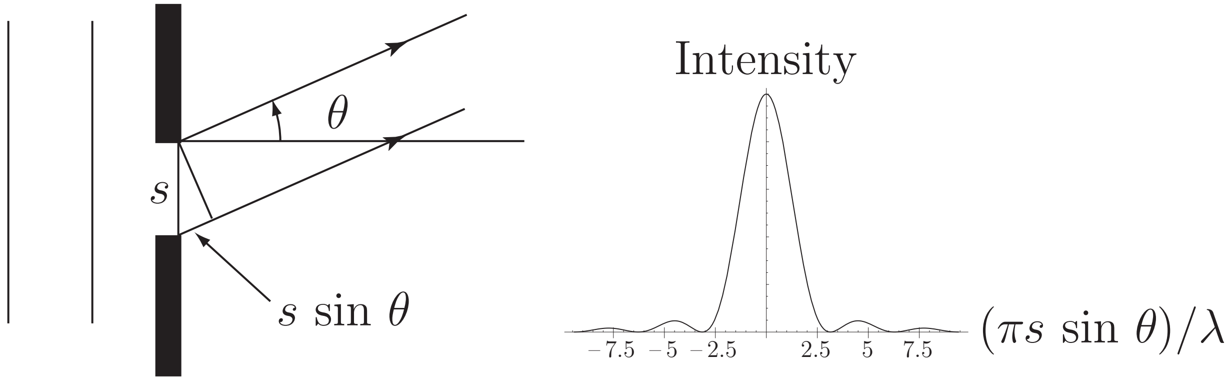
- Plane-wave limit
- Criterion for destructive interference (minimum of the diffraction pattern):

$$s \sin \theta \approx m\lambda$$

For small angles ($\theta \ll 1$), and for the first minimum,

$$\theta \approx \frac{\lambda}{s}$$

- Fraunhofer diffraction by a slit of width s:

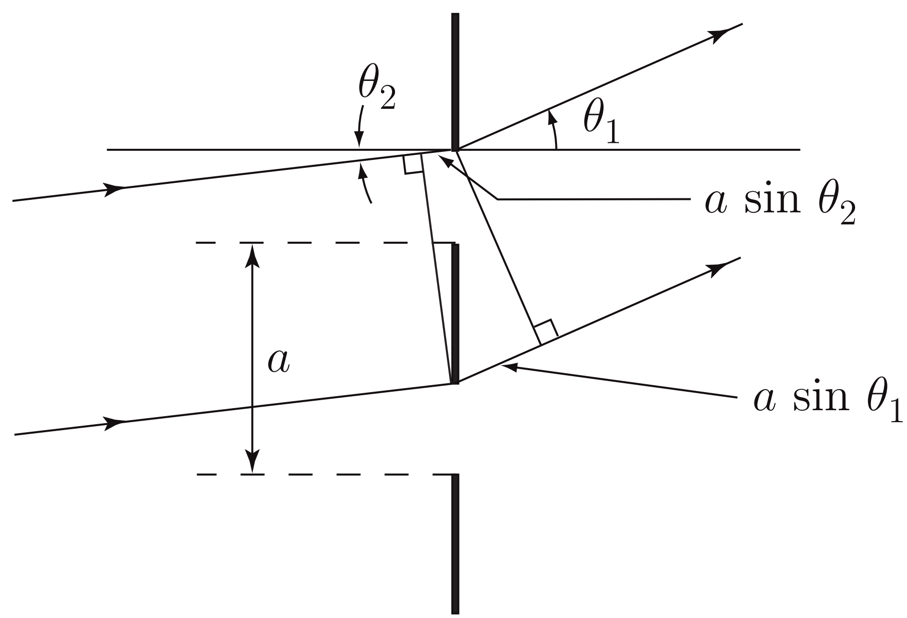


DIFFRACTION GRATINGS (PHYSICS 2 APPROACH)

- The grating shown below consists of a repeating pattern of opaque screen + aperture, with period a
- The path difference for parallel rays through adjacent apertures is

$$\Delta L = a \sin\theta_1 - a \sin\theta_2$$

- Constructive interference occurs when $\boxed{\Delta L = m\lambda}$ (where m = integer)



ORDERS OF DIFFRACTION

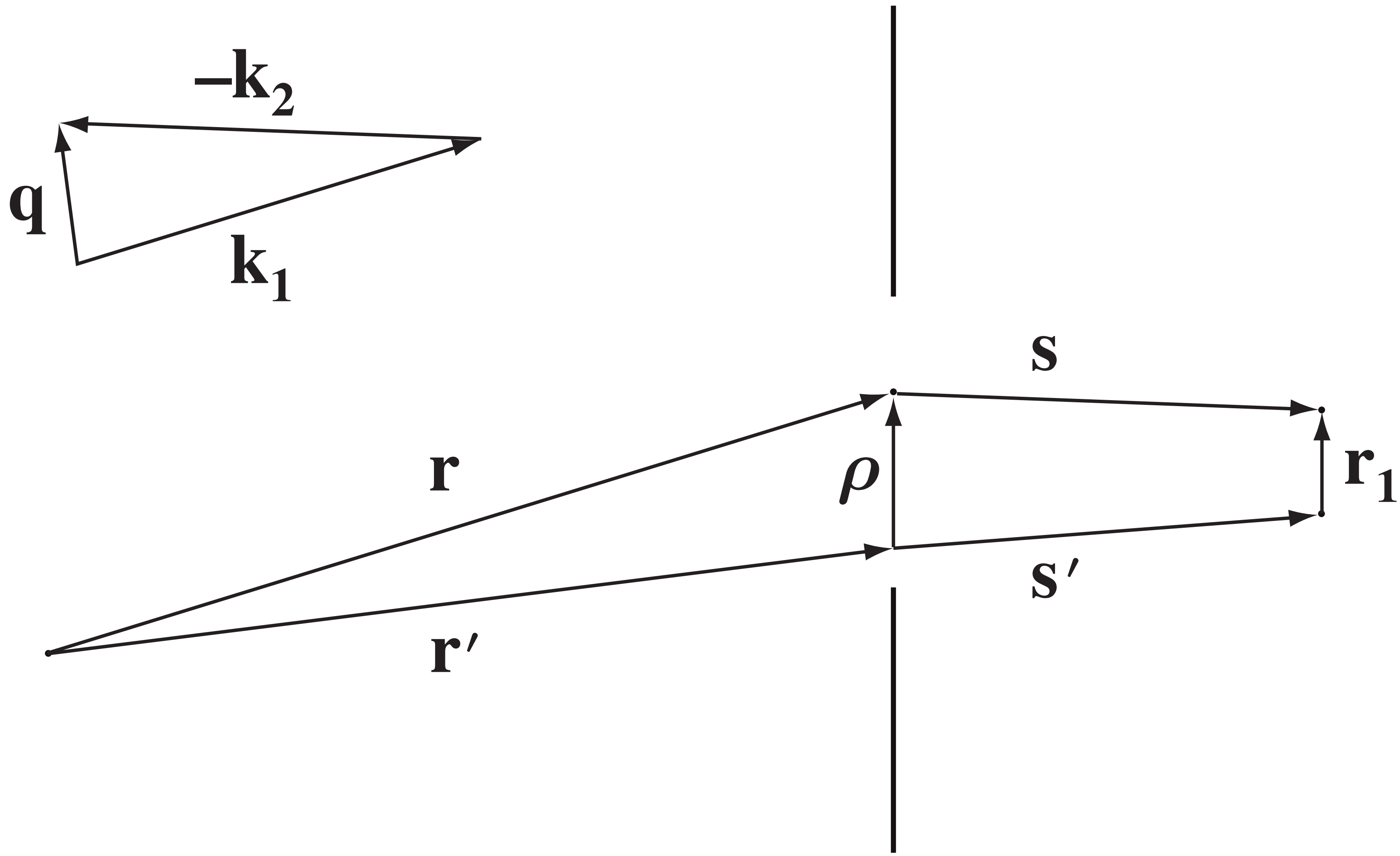
- Constructive interference occurs when $\boxed{\Delta L = m\lambda}$
- The integer $|m|$ is called the **order of diffraction**
  - ▷ The zero order ($m = 0$) corresponds to transmission straight through the grating
  - ▷ The wavelength interval $\Delta\lambda_{\mathrm{FSR}}$ between successive orders of diffraction that exactly overlap (have the same θ_1 and θ_2), such that

$$\lambda_m = a\sin\theta_1 - a\sin\theta_2 = \lambda_{m+1} + \Delta\lambda_{\mathrm{FSR}},$$

is called the **free spectral range** of the grating:

$$\Delta\lambda_{\mathrm{FSR}} = \frac{\lambda_{m+1}}{m} \approx \frac{\lambda}{m}$$

COORDINATES FOR THE DIFFRACTION INTEGRAL



DIFFRACTION GRATINGS (1)

- Fraunhofer diffraction by a screen:

$$E(\mathbf{r}_1) = C \int_S \tau(\boldsymbol{\rho})\, e^{i\mathbf{q}\cdot\boldsymbol{\rho}}\, dS$$

where $\mathbf{q} := \mathbf{k}_1 - \mathbf{k}_2$, $\mathbf{k}_1 := k\hat{\mathbf{r}}'$, $\mathbf{k}_2 := k\hat{\mathbf{s}}'$ (see preceding slide), τ is the transmission function of the screen, $\boldsymbol{\rho}$ is the position vector in the screen, and C is a constant

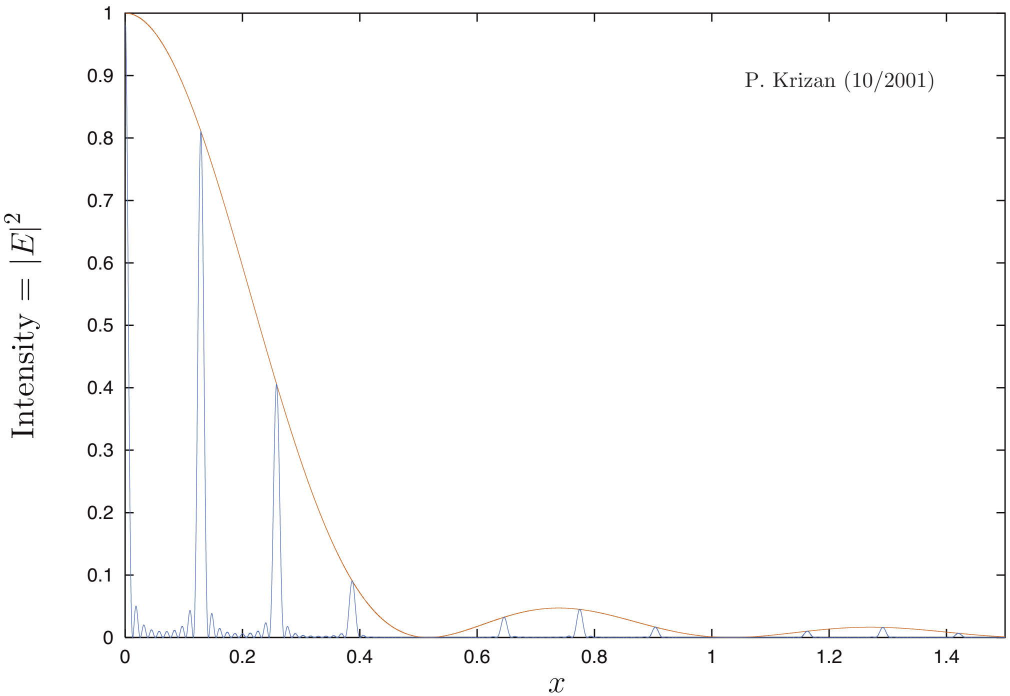
  - ▷ For our simple diffraction-grating model, $\tau = 0$ in the opaque regions, and $\tau = 1$ in the transparent regions
  - ▷ Fraunhofer diffraction by a screen with N identical apertures (a grating!):
    - ◦ Aperture A_n is displaced from aperture A_1 by a vector $\mathbf{a}_n = (n-1)\mathbf{a}$
    - ◦ Amplitude of diffracted field is

$$E(\mathbf{r}_1) = C \int_S \tau(\boldsymbol{\rho})\, e^{i\mathbf{q}\cdot\boldsymbol{\rho}}\, dS = \sum_{n=1}^{N} e^{i\mathbf{q}\cdot\mathbf{a}_n} \int_{A_1} e^{i\mathbf{q}\cdot\boldsymbol{\rho}}\, dS$$

DIFFRACTION GRATINGS (2)

- Amplitude of field diffracted by a screen with N identical, periodically arranged apertures:

$$E(\mathbf{r}_1) = \underbrace{\sum_{n=1}^{N} e^{i\mathbf{q}\cdot\mathbf{a}_n}}_{\text{multiple-beam interference pattern}} \times \underbrace{\int_{A_1} e^{i\mathbf{q}\cdot\boldsymbol{\rho}}\, dS}_{\text{diffraction pattern of one aperture}}$$

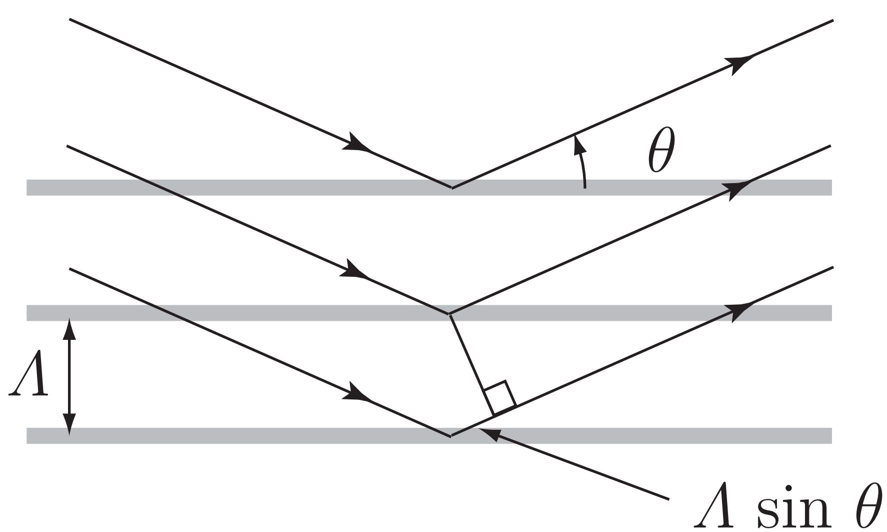


BRAGG REFLECTION

- Plane-wave limit
- **Bragg's law** for constructive interference of plane waves reflected from a refractive-index grating:

$$2\Lambda \sin\theta = m\lambda/\bar{n}$$

  - ▷ Λ is the grating period, m is an integer, and $\bar{n}$ is the mean refractive index of the medium



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