OVERVIEW OF GEOMETRICAL OPTICS

- Transition from Maxwell's equations to ray optics
- Ray optics in media with piecewise constant refractive index
- Imaging systems
  - ▷ Gaussian optics
  - ▷ Thin lenses
  - ▷ Thick lenses
  - ▷ Optical instruments
- Geometrical theory of third-order aberrations

GEOMETRICAL-OPTICS LIMIT OF MAXWELL'S EQUATIONS (1)

- Let $\mathbf{E} = \mathbf{e}(\mathbf{r})e^{ik_0(\mathcal{S}(\mathbf{r})-ct)}$ and $\mathbf{H} = \mathbf{h}(\mathbf{r})e^{ik_0(\mathcal{S}(\mathbf{r})-ct)}$; $\mathcal{S}$ is the **eikonal**
  - ▷ For example, for a plane wave propagating in a direction $\hat{\mathbf{s}}$,

$$\mathcal{S}(\mathbf{r}) = \hat{\mathbf{s}} \cdot \mathbf{r}$$

- Maxwell's equations imply that

$$\mathbf{e} \cdot (\nabla \mathcal{S}) = -\frac{1}{ik_0}\left[\mathbf{e} \cdot \nabla(\ln \epsilon) + \nabla \cdot \mathbf{e}\right] e^{ik_0 \mathcal{S}(\mathbf{r})}$$

$$\mathbf{h} \cdot (\nabla \mathcal{S}) = -\frac{1}{ik_0}\left[\mathbf{h} \cdot \nabla(\ln \mu) + \nabla \cdot \mathbf{h}\right] e^{ik_0 \mathcal{S}(\mathbf{r})}$$

$$(\nabla \mathcal{S}) \times \mathbf{e} - c\mu \mathbf{h} = -\frac{1}{ik_0}\nabla \times \mathbf{e}$$

$$(\nabla \mathcal{S}) \times \mathbf{h} + c\epsilon\, \mathbf{e} = -\frac{1}{ik_0}\nabla \times \mathbf{h}$$

- In the geometrical-optics (short-wavelength) limit, $\frac{2\pi}{\lambda_0} = k_0 \longrightarrow \infty$, so the right-hand sides can all be set to zero

GEOMETRICAL-OPTICS LIMIT OF MAXWELL'S EQUATIONS (2)

- The short-wavelength limit is a version of the slowly-varying-envelope approximation
  - ▷ For example,

$$|(ik_0)^{-1}\nabla\cdot\mathbf{e}| \sim O((k_0 l)^{-1}|\mathbf{e}|) \sim O((\lambda_0/l)|\mathbf{e}|),$$

    where l is a distance in which the envelope $\mathbf{e}$ or a major component of $\mathbf{e}$ changes by a factor of 2
  - ▷ If $\lambda_0/l \ll 1$, then

$$|(ik_0)^{-1}\nabla\cdot\mathbf{e}| \ll |\mathbf{e}|$$

    which justifies neglecting $(ik_0)^{-1}\nabla\cdot\mathbf{e}$ in comparison with terms of the same order as $\mathbf{e}$

GEOMETRICAL-OPTICS LIMIT OF MAXWELL'S EQUATIONS (3)

- Short-wavelength limit of Maxwell's equations:

$$\mathbf{e} \cdot (\nabla \mathcal{S}) = 0$$

$$\mathbf{h} \cdot (\nabla \mathcal{S}) = 0$$

$$(\nabla \mathcal{S}) \times \mathbf{e} = c\mu \mathbf{h}$$

$$(\nabla \mathcal{S}) \times \mathbf{h} = -c\epsilon\, \mathbf{e}$$

▷ Eliminate **h** and use the BAC-CAB identity:

$$(\nabla \mathcal{S}) \times \mathbf{h} = \frac{1}{c\mu} \{(\nabla S)\,[\mathbf{e} \cdot (\nabla S)] - \mathbf{e}\,[(\nabla S) \cdot (\nabla S)]\} = -\frac{1}{c\mu} (\nabla S)^2 \mathbf{e} = -c\epsilon\, \mathbf{e}$$

▷ The result is the **eikonal equation**

$$\boxed{|\nabla \mathcal{S}|^2 = \mu\epsilon c^2 = \mu_r \epsilon_r = n^2}$$

where n is the **refractive index**

WAVEFRONTS

- Geometrical-optics limit of Maxwell's equations
  - ▷ Eikonal equation: $|\nabla \mathcal{S}|^2 = n^2$
  - ▷ The surfaces defined by the equation $\boxed{\mathcal{S}(\mathbf{r}) = \text{constant}}$ are the **geometrical wave fronts**
  - ▷ $\boxed{\hat{\mathbf{s}}(\mathbf{r}) = n^{-1}\nabla \mathcal{S}(\mathbf{r})}$ is normal (perpendicular) to the wavefront at the point $\mathbf{r}$
  - ▷ $\hat{\mathbf{s}}$ is a unit vector because of the eikonal equation:

$$\hat{\mathbf{s}} \cdot \hat{\mathbf{s}} = n^{-2}|\nabla \mathcal{S}|^2 = 1$$

RAYS

- **Rays** are defined similarly to field lines:
  A **ray** is a curve that is tangent to some wavefront unit normal $\hat{\mathbf{s}}$ at each point $\mathbf{r}$ on the curve
  - ▷ If ds is the distance between two neighboring points on a ray, then the unit vector that is tangent to the ray is $\boxed{\hat{\mathbf{s}} = d\mathbf{r}/ds}$ (by geometry)
  - ▷ The unit tangent to a ray,
  $$n^{-1}\nabla\mathcal{S} = \hat{\mathbf{s}} = \frac{d\mathbf{r}}{ds} = \frac{dx}{ds}\hat{\mathbf{x}} + \frac{dy}{ds}\hat{\mathbf{y}} + \frac{dz}{ds}\hat{\mathbf{z}},$$
  has direction cosines dx/ds, dy/ds, dz/ds
  - ▷ Because $\nabla\mathcal{S}$ is perpendicular to the surface $\mathcal{S}$ = constant, and $\hat{\mathbf{s}}$ is parallel to $\nabla\mathcal{S}$, **rays are perpendicular to wavefronts**

RAYS AND THE EIKONAL (1)

- The equation that relates rays to the eikonal is

$$\boxed{n\frac{d\mathbf{r}}{ds} = \nabla \mathcal{S}}$$

  ▷ It turns out that a simpler equation, without the eikonal $\mathcal{S}$, results if we calculate $d/ds(nd\mathbf{r}/ds)$

  ▷ We calculate the rate of change of a vector field $\mathbf{A}$ along a ray:

$$\begin{aligned}
\frac{d\mathbf{A}}{ds} &= \frac{dA_x}{ds}\hat{\mathbf{x}} + \frac{dA_y}{ds}\hat{\mathbf{y}} + \frac{dA_z}{ds}\hat{\mathbf{z}} \\
&= \hat{\mathbf{x}}\left(\frac{\partial A_x}{\partial x}\frac{dx}{ds} + \frac{\partial A_x}{\partial y}\frac{dy}{ds} + \frac{\partial A_x}{\partial z}\frac{dz}{ds}\right) + \cdots \\
&= \hat{\mathbf{x}}\left(\frac{d\mathbf{r}}{ds}\cdot\nabla\right)A_x + \hat{\mathbf{y}}\left(\frac{d\mathbf{r}}{ds}\cdot\nabla\right)A_y + \hat{\mathbf{z}}\left(\frac{d\mathbf{r}}{ds}\cdot\nabla\right)A_z \\
&= \left(\frac{d\mathbf{r}}{ds}\cdot\nabla\right)\mathbf{A} \quad \text{in Cartesian coordinates}
\end{aligned}$$

RAYS AND THE EIKONAL (2)

- Apply the equation for $d\mathbf{A}/ds$ on the previous slide to calculate $d/ds(n d\mathbf{r}/ds)$:

$$\begin{aligned}
\frac{d}{ds}\left(n\frac{d\mathbf{r}}{ds}\right) &= \frac{d}{ds}(\nabla \mathcal{S}) \quad \text{because} \quad n\frac{d\mathbf{r}}{ds} = \nabla\mathcal{S} \\
&= \left(\frac{d\mathbf{r}}{ds}\cdot\nabla\right)(\nabla\mathcal{S}) \quad \text{because} \quad \frac{d\mathbf{A}}{ds} = \left(\frac{d\mathbf{r}}{ds}\cdot\nabla\right)\mathbf{A} \\
&= \left(\frac{1}{n}(\nabla\mathcal{S})\cdot\nabla\right)(\nabla\mathcal{S}) \quad \text{because} \quad \frac{d\mathbf{r}}{ds} = n^{-1}\nabla\mathcal{S} \\
&= \frac{1}{2n}\nabla\left[\nabla\mathcal{S}\cdot\nabla\mathcal{S}\right] \\
&= \frac{1}{2n}\nabla\left[(\nabla\mathcal{S})^2\right] \\
&= \frac{1}{2n}\nabla(n^2) \quad \text{because} \quad (\nabla\mathcal{S})^2 = n^2 \\
&= \nabla n
\end{aligned}$$

▷ This eliminates the eikonal from the equation that determines a ray

RAY EQUATION

- Ray equation that does not involve the eikonal $\mathcal{S}$:

$$\frac{d}{ds}(n\hat{\mathbf{s}}) = \nabla n$$

where s = distance measured along the ray (arc length) and
$\hat{\mathbf{s}}$ = unit tangent vector to the ray

▷ This equation determines a ray by determining the tangent vector at each point along the ray

▷ Example: n = constant

- $\nabla n = \mathbf{0} \Rightarrow \hat{\mathbf{s}}$ = constant vector = $\mathbf{a}$
- Then $d\mathbf{r}/ds = \mathbf{a}$ = constant vector $\Rightarrow \mathbf{r} = \mathbf{a}s + \mathbf{b}$ (a straight line!)
- Different rays can have different vectors $\mathbf{a}$ and $\mathbf{b}$
- **In a medium with constant refractive index, rays are straight lines**

IDEALIZED WAVEFRONTS AND RAYS

- Plane wavefronts
  - ▷ Rays are parallel lines
  - ▷ Special case of a spherical wavefront
- Spherical wavefronts
  - ▷ Rays are are straight lines that converge to, or diverge from, a point focus
  - ▷ An extended incoherent source can be modeled as a collection of point sources
  - ▷ At large distances from a focus, the wavefronts produced by a laser in the lowest transverse mode are approximately spherical
- Cylindrical wavefronts
  - ▷ Rays are straight lines that converge to, or diverge from, a line focus

GEOMETRICAL OPTICS

- All media are assumed to have piecewise-constant refractive indices
  - ▷ The media of which optical elements such as lenses are made are homogeneous and isotropic
  - ▷ Rays are straight-line segments that change direction only at the interfaces between media
- Ray directions are determined by Snell's laws and the geometry of optical elements (lenses, mirrors, etc.)
  - ▷ In the geometrical-optics approximation, imaging problems can be solved using only geometry
  - ▷ Geometrical optics is useful at the start of most optical designs, but perhaps not in the final stages

SNELL'S LAWS

- The normal to a dielectric interface and the incident ray define the **plane of incidence**
  - ▷ The incident (i), reflected (r) and refracted (transmitted) (t) rays are all in the plane of incidence
  - ▷ Snell's law of reflection:

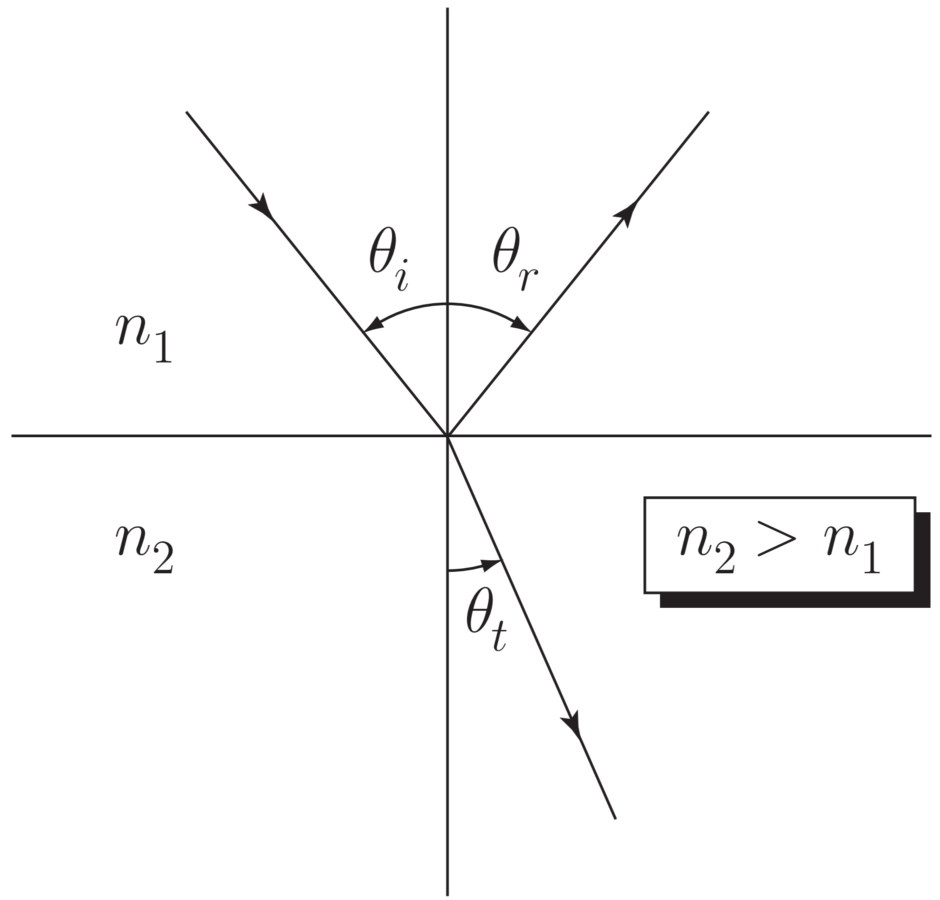
$$\theta_r = \theta_i$$

  - ▷ Snell's law of refraction:

$$n_2 \sin\theta_t = n_1 \sin\theta_i$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1} = \frac{v_1}{v_2}$$

ANGLES FOR SNELL'S LAWS



IMAGING SYSTEMS

- Lenses and mirrors
  - ▷ Form an image of the source
  - ▷ Used for:
    - ◦ Collimating light from an extended source or a laser
    - ◦ Focusing light into fibers and waveguides
    - ◦ Forming images in optical instruments
      - ◇ Telescopes
      - ◇ Microscopes
      - ◇ Projection systems
- Design by ray tracing
  - ▷ Initial sketch may be a hand-drawn ray trace using Gaussian optics
  - ▷ Final design uses computer-generated ray tracing and optimization
- Physical (wave) optics is necessary in designing to tolerances below $\lambda/4$

GAUSSIAN OPTICS (1)

- Consider optical systems that have an axis of **rotational symmetry** (Z)
  - ▷ For now, consider just the $X - Z$ plane
  - ▷ Rays are straight lines
    - ∘ Can be specified with intercept and slope
    - ∘ Most convenient choice: Specify

$$\boxed{x = \text{height at } z}$$

and

$$\boxed{p = n \sin\alpha = n\hat{\mathbf{s}} \cdot \hat{\mathbf{x}}}$$

where α = angle ray makes with Z-axis and n = refractive index

GAUSSIAN OPTICS (2)

- **Paraxial rays** are rays such that $\alpha \ll 1$
- In the **paraxial approximation**, also known as **Gaussian optics**,

$$\sin\alpha \approx \alpha$$

and therefore

$$p \approx n\alpha$$

- ▷ Corrections to the paraxial approximation are called **aberrations**
- ▷ The next approximation better than the paraxial approximation is

$$\sin\alpha \approx \alpha - \frac{\alpha^3}{3!}$$

which leads to "third-order theory" and the so-called Seidel aberrations (spherical aberration, coma, astigmatism, curvature of field, and distortion)

STRAIGHT-LINE PROPAGATION (1)

- Exact equations for propagation from $z = z_0$ to $z = z_1$:

$$x_1 = x_0 + (z_1 - z_0) \tan \alpha$$

$$p_1 = p_0$$

- Paraxial (Gaussian) approximation:

$$x_1 = x_0 + \frac{z_1 - z_0}{n} p_0$$
$$p_1 = p_0 = n\alpha$$

STRAIGHT-LINE PROPAGATION (2)

- Matrix form of the Gaussian equations for straight-line propagation:

$$\begin{pmatrix} x_1 \\ p_1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{z_1 - z_0}{n} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ p_0 \end{pmatrix}$$

- In general, for the Gaussian (paraxial) description of an optical system,

$$\begin{pmatrix} x_1 \\ p_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x_0 \\ p_0 \end{pmatrix}$$

where $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is called the **ray matrix**

  ▷ The ray matrix for straight-line propagation from $z = z_0$ to $z = z_1$ is

$$\mathbf{M}_{01} = \begin{pmatrix} 1 & \frac{z_1 - z_0}{n} \\ 0 & 1 \end{pmatrix}$$

  ▷ This ray matrix has the property that $\det [\mathbf{M}_{01}] = 1$

PROPAGATION ACROSS A FLAT INTERFACE

- By rotational symmetry, a flat interface must be perpendicular to Z
  - ▷ Assume refractive indices n_0 to the left of the interface and n_1 to the right
- Snell's law of refraction:

$$n_1 \sin \alpha_1 = n_0 \sin \alpha_0$$

Then

$$x_1 = x_0$$

and

$$p_1 = p_0$$

- For propagation across a flat interface, the ray matrix is the identity matrix:

$$\mathbf{M}_{01} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

SPHERICAL REFRACTING INTERFACE (1)

- By rotational symmetry, the center is on the Z axis
  - ▷ Assume refractive indices n_0 to the left of the interface and n_1 to the right
  - ▷ Convention: Radius of curvature $= R > 0$ if center is to the right
- Exact relations:

$$x_1 = x_0 = R \sin \theta$$

$$n_1 \sin \theta_1 = n_0 \sin \theta_0$$

where

$$\theta_0 = \theta + \alpha_0, \quad \theta_1 = \theta + \alpha_1$$

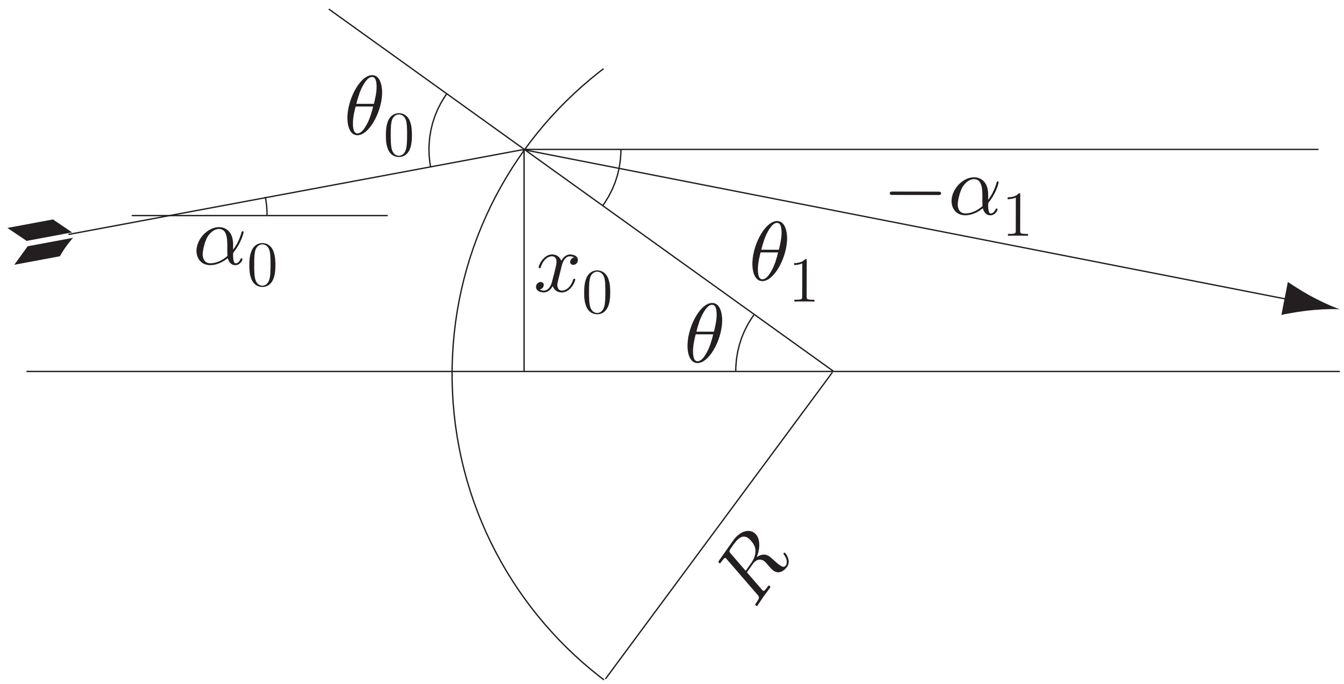
- In Gaussian optics, $\sin \theta = \theta$, $\sin(\theta + \alpha_0) = \theta + \alpha_0$, etc.
- Gaussian relation:

$$n_1 \alpha_1 = n_0 \alpha_0 - (n_1 - n_0) \frac{x_0}{R}$$

Then

$$p_1 = p_0 - (n_1 - n_0) \frac{x_0}{R}$$

SPHERICAL REFRACTING SURFACE



SPHERICAL REFRACTING INTERFACE (2)

- Ray-matrix equation for a ray that has just passed through a spherical refracting surface:

$$\begin{pmatrix} x_1 \\ p_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ p_0 \end{pmatrix}$$

where

$$P = \frac{n_1 - n_0}{R}$$

is the **power** of the refracting surface

▷ The units of refractive power are **diopters**

▷ Once again, the determinant of the ray matrix

$$\mathbf{M}_{01} = \begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix}$$

is

$$\det\left[\mathbf{M}_{01}\right] = 1$$

THIN LENS

- Model as two spherical refracting surfaces at the same position z on the optical axis, with radii of curvature R_1 and $-R_2$

$$\begin{pmatrix} x_1 \\ p_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -P_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -P_1 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ p_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -(P_1 + P_2) & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ p_0 \end{pmatrix}$$

where

$$P_1 = \frac{n_1 - n_0}{R_1}, \quad P_2 = -\frac{n_1 - n_0}{R_2}$$

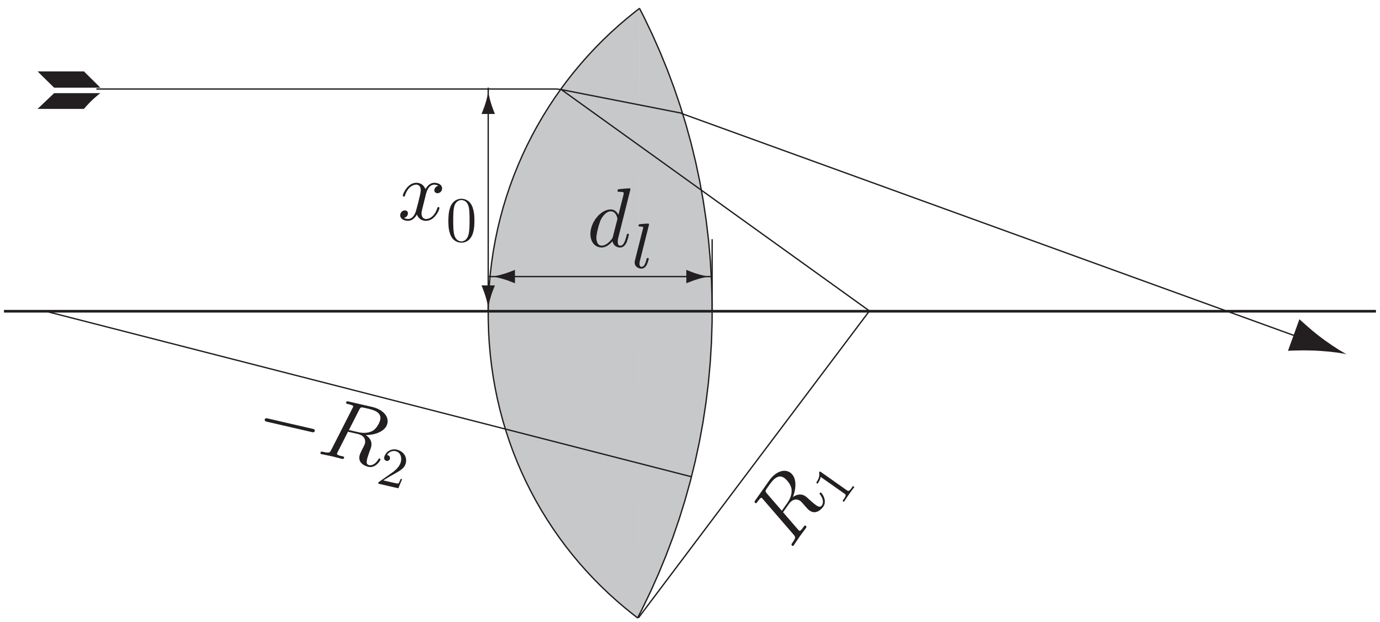
and

$$P_1 + P_2 = (n_1 - n_0)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{n_0}{f}$$

▷ We shall see that f is the **focal length** of the lens

▷ $P = n_0/f$ is the (refractive) power of the lens

LENS PARAMETERS



FOCUSING BY A THIN LENS (1)

- Consider a lens at z_1 and a focal point at z_2
- Traverse the lens and translate through $f = z_2 - z_1$ to go from the lens to the focal point, remembering that $d_l = 0$ for a thin lens:

$$\begin{pmatrix} x_2 \\ p_2 \end{pmatrix} = \begin{pmatrix} 1 & f/n_0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ p_1 \end{pmatrix} = \begin{pmatrix} 1 - P\dfrac{f}{n_0} & \dfrac{f}{n_0} \\ -P & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ p_1 \end{pmatrix}$$

  - ▷ z_2 is the location of the focal plane of the lens if all rays that pass through (x_1, z_1) with the same value of p_1 also pass through (x_2, z_2), regardless of the value of x_1
  - ▷ Since x_2 must be independent of x_1:

$$1 - P\frac{f}{n_0} = 0 \quad \text{and therefore} \quad \boxed{\frac{n_0}{f} = \frac{1}{P} = (n_1 - n_0)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

FOCUSING BY A THIN LENS (2)

- The relation

$$\frac{n_0}{f} = (n_1 - n_0)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

  is known as the **lensmaker's equation** for a thin lens

- There's also useful information in the equation for x_2:

$$x_2 = \frac{p_1}{n_0} f$$

  ▷ This equation tells us that:
  - A bundle of parallel rays with $\sin\alpha_1 = p_1/n_0$ incident on a thin lens located at z_1 is focused to a single point in a focal plane at $z = z_1 + f$
  - The displacement of the focal point from the optical axis, x_2, is uniquely determined by the angle α_1 of the incident bundle of rays
  - In Gaussian optics, the focal region is flat (no curvature of field)

IMAGING BY A THIN LENS (1)

- Consider an object at z_0, a lens at z_1, and an image at z_2
- Translate through $d_o = z_1 - z_0$ to go from the object to the lens, traverse the lens, and translate through $d_i = z_2 - z_1$ to go from the lens to the image:

$$\begin{pmatrix} x_2 \\ p_2 \end{pmatrix} = \begin{pmatrix} 1 & d_i/n_0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix} \begin{pmatrix} 1 & d_o/n_0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ p_0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - P\dfrac{d_i}{n_0} & \dfrac{d_i}{n_0} + \dfrac{d_o}{n_0} - P\dfrac{d_i d_o}{n_0^2} \\ -P & 1 - P\dfrac{d_o}{n_0} \end{pmatrix} \begin{pmatrix} x_0 \\ p_0 \end{pmatrix}$$

▷ z_2 is the location of the image of the object at z_0 if all rays that pass through (x_0, z_0) also pass through (x_2, z_2), regardless of the value of p_0

▷ Since x_2 must be independent of p_0:

$$\frac{d_i}{n_0} + \frac{d_o}{n_0} - P\frac{d_i d_o}{n_0^2} = 0$$

IMAGING BY A THIN LENS (2)

- The imaging relation

$$\frac{d_i}{n_0} + \frac{d_o}{n_0} - P\frac{d_i d_o}{n_0^2} = 0$$

and the focal-length relation

$$P = \frac{n_0}{f}$$

imply that

$$d_i + d_o = \frac{d_i d_o}{f}$$

The **object-image relation** is

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$$

- **Newton's equation**

$$\boxed{(d_o - f)(d_i - f) = f^2}$$

gives the same information in a different, also useful form

IMAGING BY A THIN LENS (3)

- The **linear magnification** of a lens is the ratio of lateral image size to object size:

$$M = -\frac{x_2}{x_0} = -1 + P\frac{d_i}{n_0} = -1 + \frac{d_i}{f}$$

$$\boxed{M = \frac{d_i}{d_o}}$$

  ▷ The $-$ sign indicates that the image is inverted

- The **angular magnification** is, from the ray matrix,

$$-\frac{p_2}{p_1} = -1 + P\frac{d_o}{n_0} = \frac{1}{M}$$

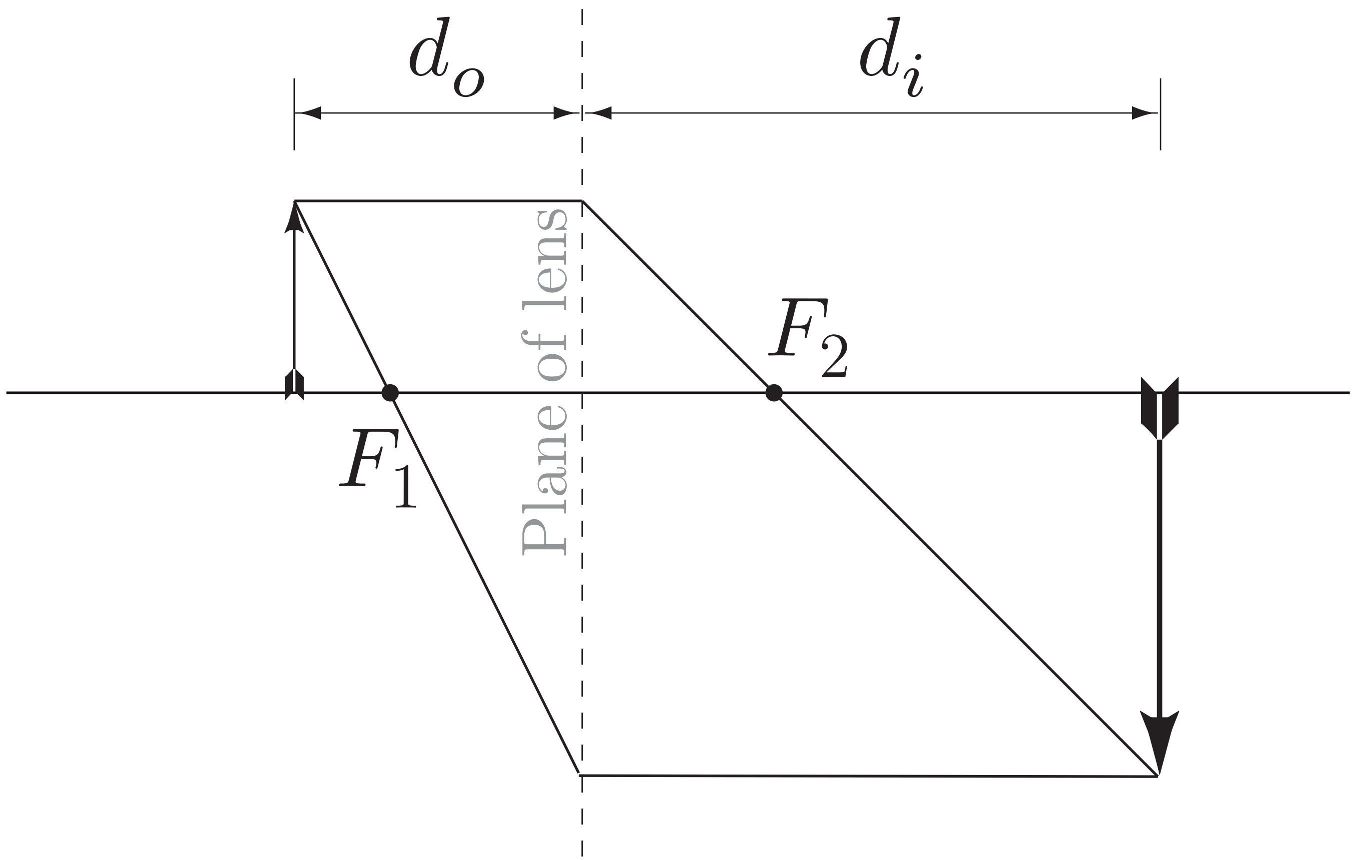
- The ray matrix of a thin lens in the image plane at $z = z_2$ is

$$\mathbf{M}_{02} = \begin{pmatrix} -M & 0 \\ -n_0/f & -1/M \end{pmatrix}$$

THIN-LENS RAY TRACING

- Begin by drawing the optical axis, a perpendicular line to indicate the plane of the lens, and the right and left focal points
- The next steps involve drawing two **principal rays** of the optical system
  - ▷ Draw a principal ray that goes through the left principal focus to the plane of the lens and then runs parallel to the optical axis
  - ▷ Draw a second principal ray that runs parallel to the optical axis from the tip of the object to the plane of the lens, and then goes through the right principal focus
  - ▷ The intersection of the two principal rays locates the tip of the image
- This procedure is valid only for paraxial rays

THIN-LENS RAY TRACE

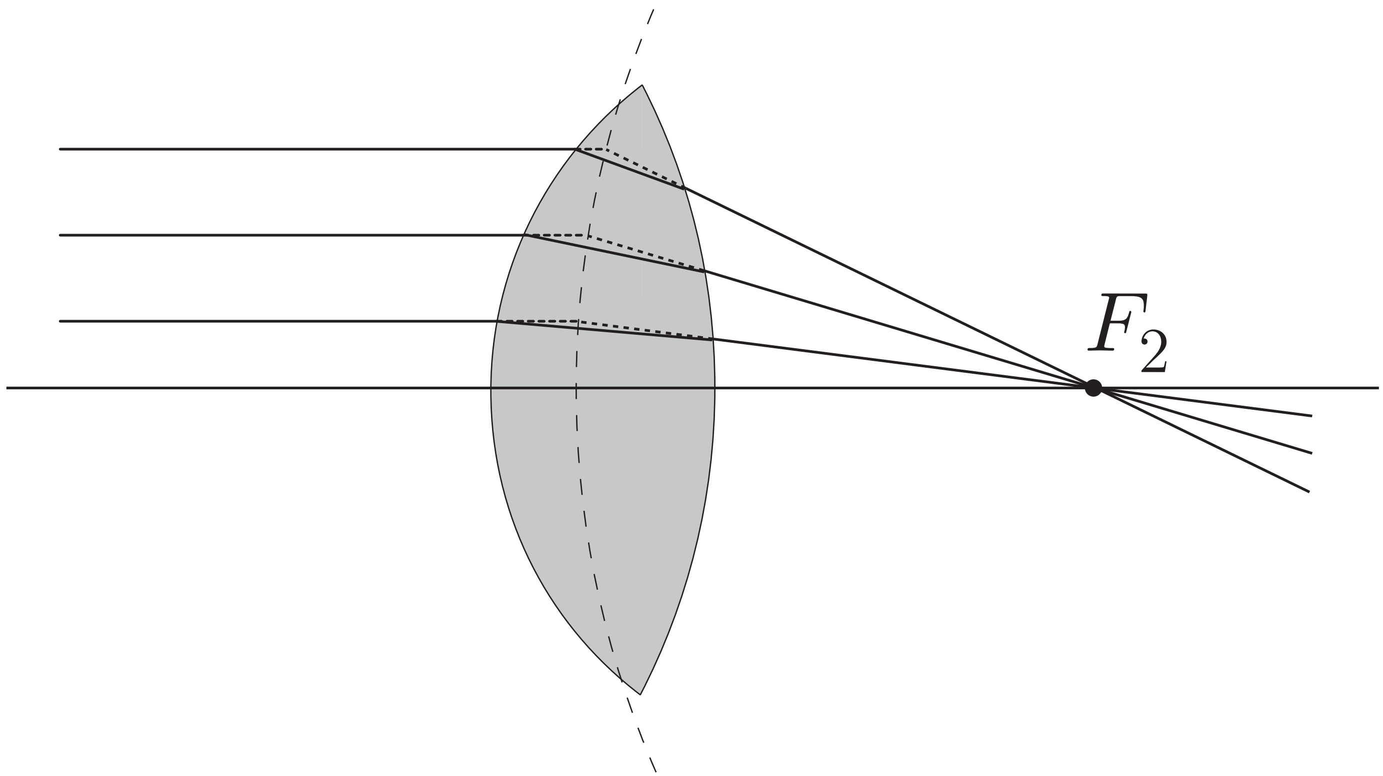


F_1, F_2: principal foci

THICK LENSES (1)

- The **principal planes** are the surfaces defined by the intersections of the incoming and outgoing segments of each ray, extended to meet
  - ▷ In general, the principal "planes" are actually curved surfaces
  - ▷ In the paraxial approximation, the principal planes are flat
- In the paraxial approximation, a thick lens (or any optical system that has an optical axis) can be described completely by its six **cardinal points**:
  - ▷ The two principal foci
  - ▷ The two **principal points**, defined as the intersections of the principal planes with the optical axis
  - ▷ The two **nodal points**, defined as the intersections with the optical axis of the (extended) incoming and outgoing segments of a ray that goes through the optical center of the lens

PRINCIPAL PLANE



Secondary principal plane

Conceptual drawing (not an accurate ray trace)

THICK LENSES (2)

- The distance h_1 from the primary principal plane to the on-axis entrance point of the lens, and the distance h_2 from the secondary principal plane to the on-axis exit point, are:

$$h_1 = -\frac{f(n_1 - 1)d_l}{R_2 n_1}, \qquad h_2 = -\frac{f(n_1 - 1)d_l}{R_1 n_1}$$

- Each paraxial focal point of a thick lens is located at a distance f from the nearer of the two principal planes
- The lensmaker's equation for a thick lens immersed in air $(n_0 = 1)$ is

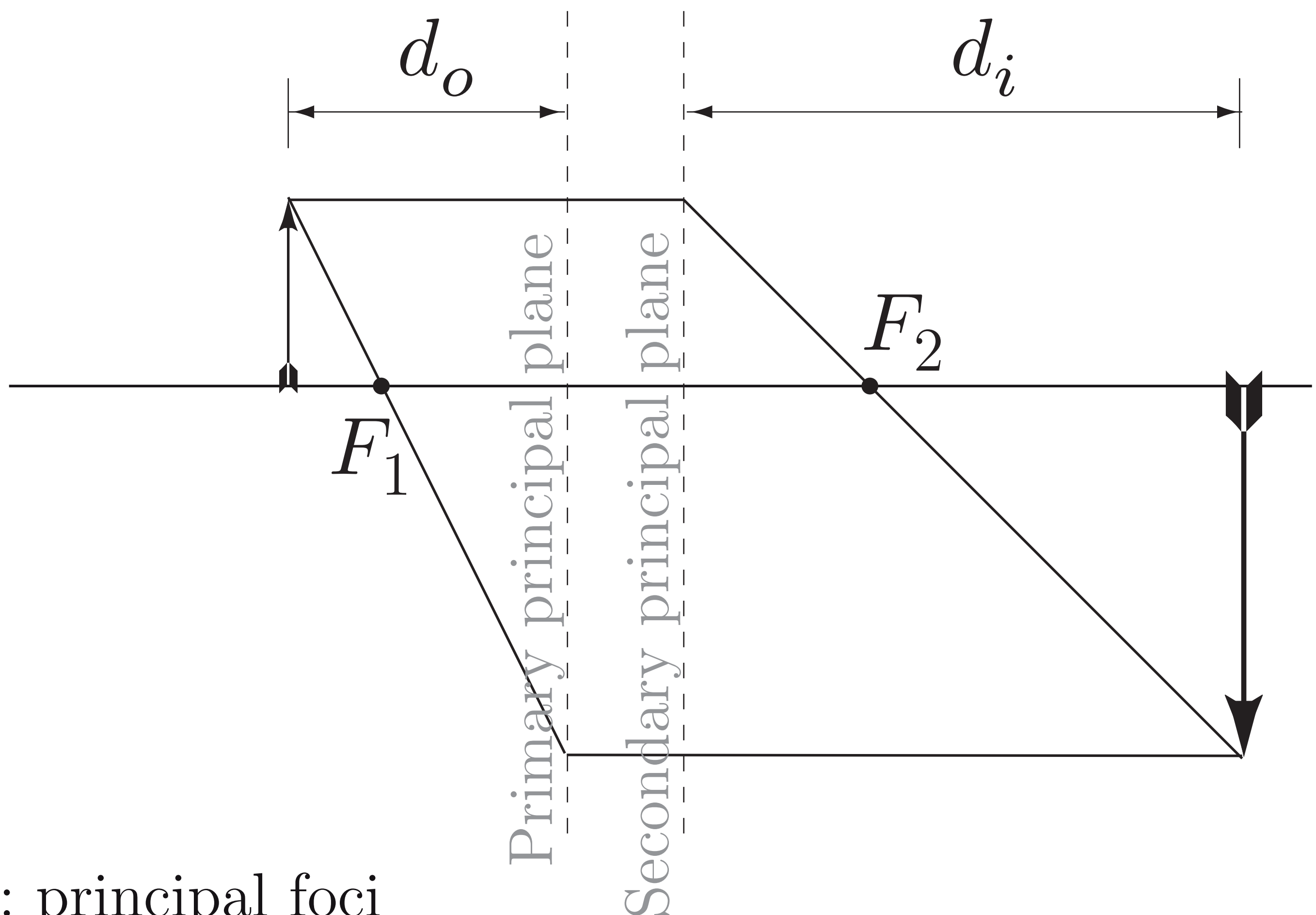
$$\boxed{\frac{1}{f} = (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_1 - 1)d_l}{n_1 R_1 R_2}\right)}$$

- The paraxial object-image relation and Newton's equation still hold if the object and image distances d_o, d_i are measured from the principal planes

THICK-LENS RAY TRACING

- Begin by drawing the optical axis, two perpendicular lines to indicate the principal planes, and the right and left focal points
- The next steps involve drawing two principal rays of the optical system
  - ▷ Draw a principal ray that goes through the left principal focus to the primary (oject-side) principal plane and then runs parallel to the optical axis
  - ▷ Draw a second principal ray that runs parallel to the optical axis from the tip of the object to the secondary (image-side) principal plane, and then goes through the right principal focus
  - ▷ The intersection of the two principal rays locates the tip of the image
- This procedure is valid only for paraxial rays

THICK-LENS RAY TRACE



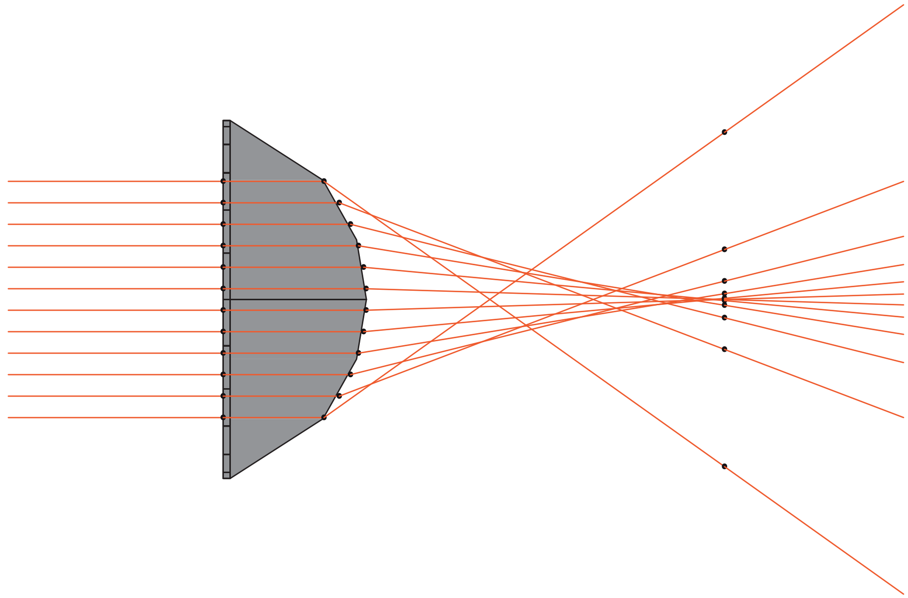
F_1, F_2: principal foci

ABERRATIONS

- Corrections to the paraxial approximation are called **aberrations**
  - ▷ Chromatic aberration: Separated images are formed for each object color
  - ▷ Monochromatic aberrations
    - ○ Some cause the image of a point to be spread out laterally and longitudinally
    - ○ Others are global defects of images
- The Seidel (monochromatic, third-order) aberrations are:
  - ▷ Spherical aberration: A point is imaged to a line
  - ▷ Coma: A point is imaged to a set of expanding, overlapping circles
  - ▷ Astigmatism: A point is imaged to a separated pair of lines
  - ▷ Curvature of field: The image of a laterally extended object is formed on a curved surface, not on a plane
  - ▷ Distortion: The lateral magnification depends on the distance of the object from the axis

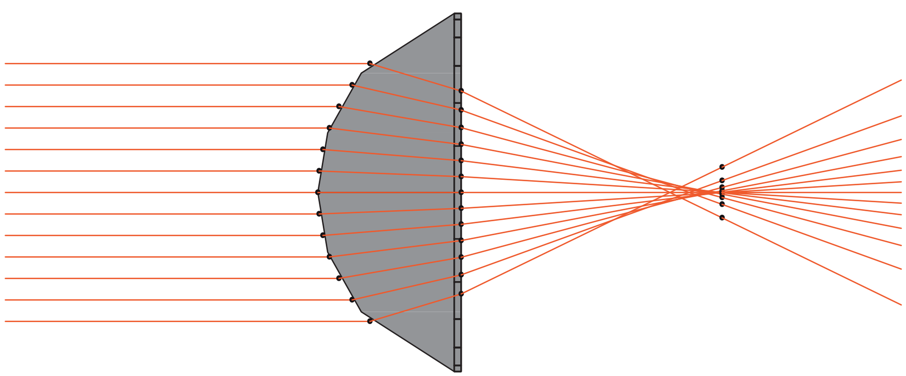
SPHERICAL ABERRATION (1)

- For a point object on-axis, or a bundle of rays parallel to the axis, the only monochromatic aberration is spherical aberration
- The Optica ray trace below illustrates spherical aberration of a thick lens
  - ▷ Plano-convex lens made of BK7 glass; aperture 50 mm
  - ▷ Black dots show ray intersections with the Gaussian focal plane



SPHERICAL ABERRATION (2)

- Spherical aberration of a lens is minimized when approximately equal ray deviations occur at the two surfaces
- The Optica ray trace below should be compared with the ray trace on the previous slide
  - ▷ Same plano-convex lens made of BK7 glass; aperture 50 mm
  - ▷ Black dots show ray intersections with the Gaussian focal plane
  - ▷ Note that there is a well-defined **circle of least confusion**



TERMINOLOGY

- The **aperture stop** is the opening that determines the amount of light that reaches the image
- The **entrance pupil** is the image of the aperture stop as seen from an object point on the optical axis, through the optical elements between the object and the aperture stop
- The **chief ray** from an object point is the ray that goes through the center of the entrance pupil
- A **marginal ray** goes from an object point to the edge of the entrance pupil
- A **meridional plane** is any plane that contains the optical axis

WAVEFRONT ABERRATION

- The **wavefront aberration** ϕ is the distance between the actual wavefront and a sphere centered on the Gaussian image point
- Expressed in terms of the coordinates defined on the next slide, the contributions of the Seidel aberrations to the total wavefront aberration are:
  - ▷ Spherical aberration

$$\phi = -\tfrac{1}{4} B a^4 \rho^4$$

  - ▷ Coma

$$\phi = F y_0 a^3 \rho^3 \cos\theta$$

  - ▷ Astigmatism

$$\phi = -C y_0^2 a^2 \rho^2 \cos^2\theta$$

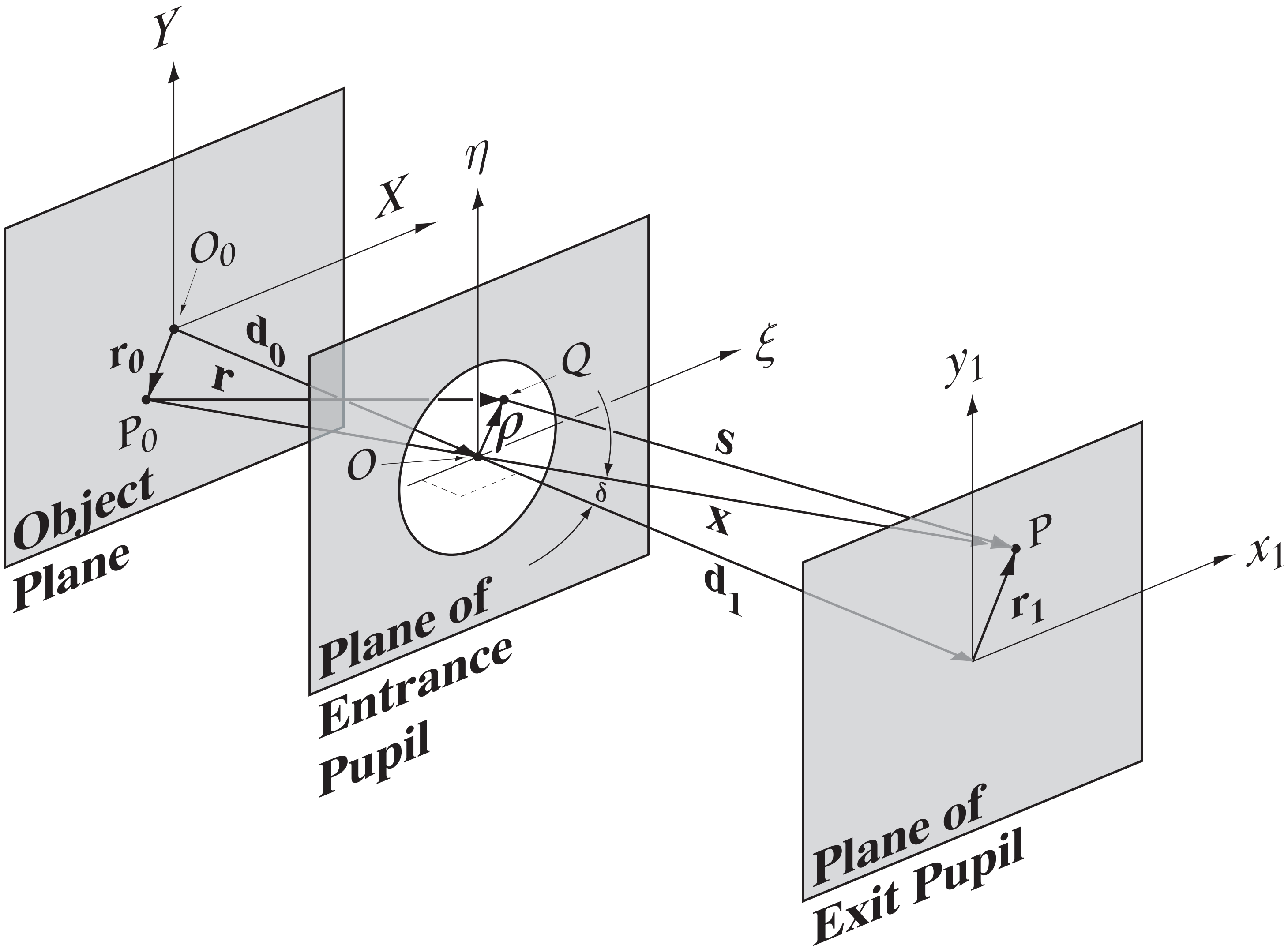
  - ▷ Curvature of field

$$\phi = -\tfrac{1}{2} D y_0^2 a^2 \rho^2$$

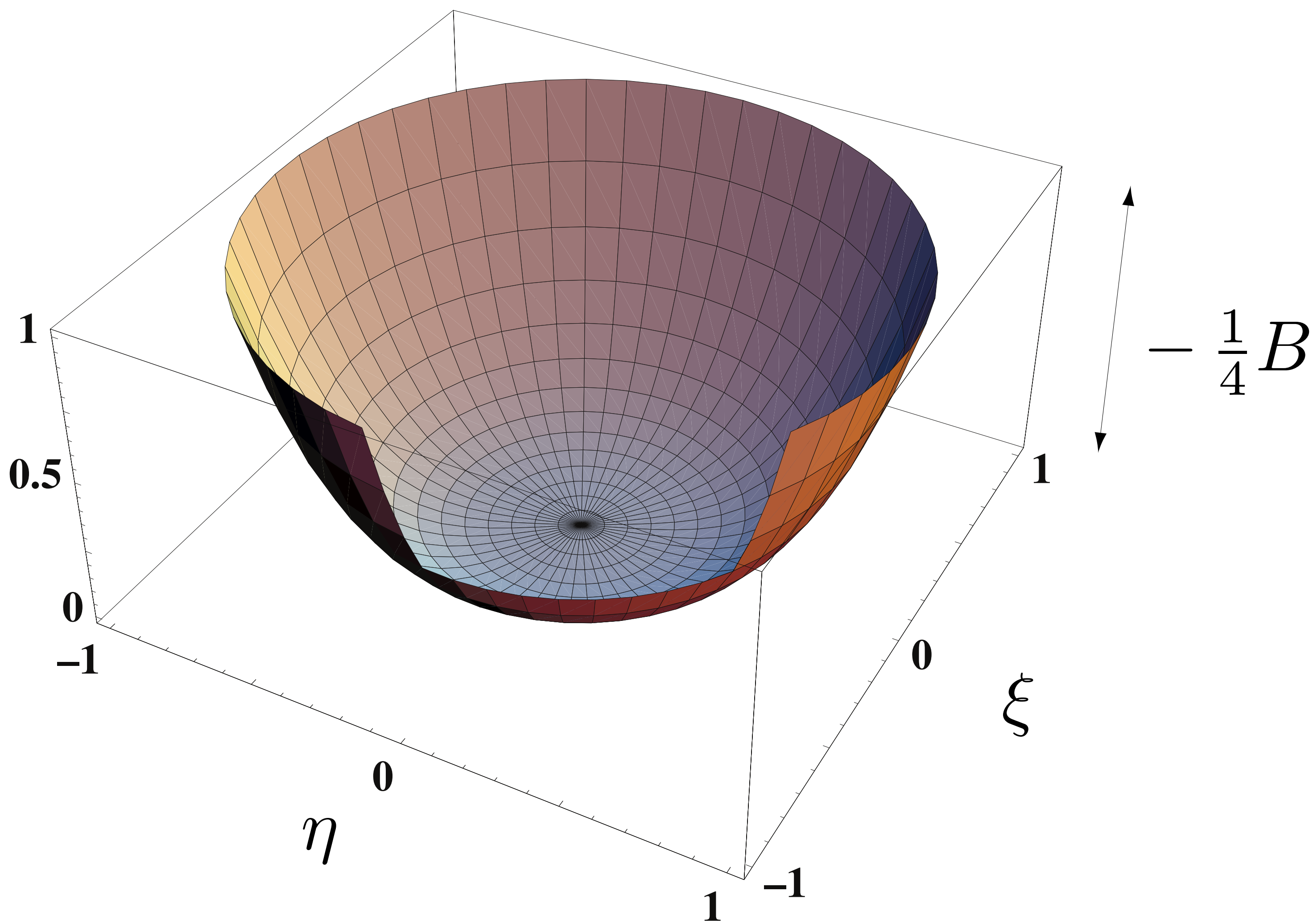
  - ▷ Distortion

$$\phi = E y_0^3 a \rho \cos\theta$$

COORDINATES FOR CALCULATION OF ABERRATIONS



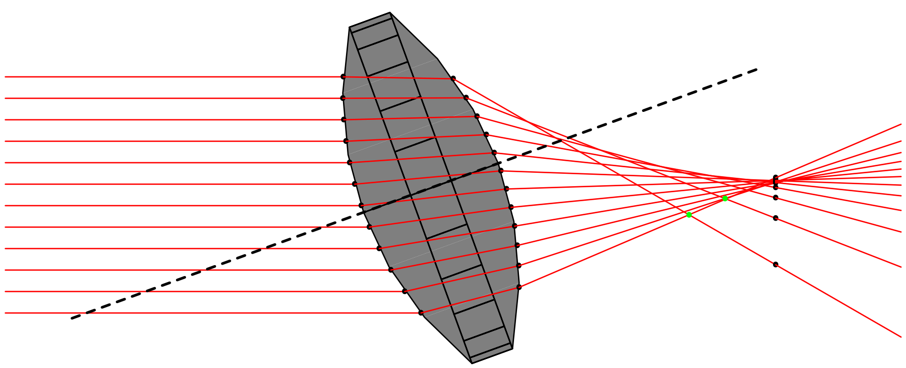
Spherical Aberration



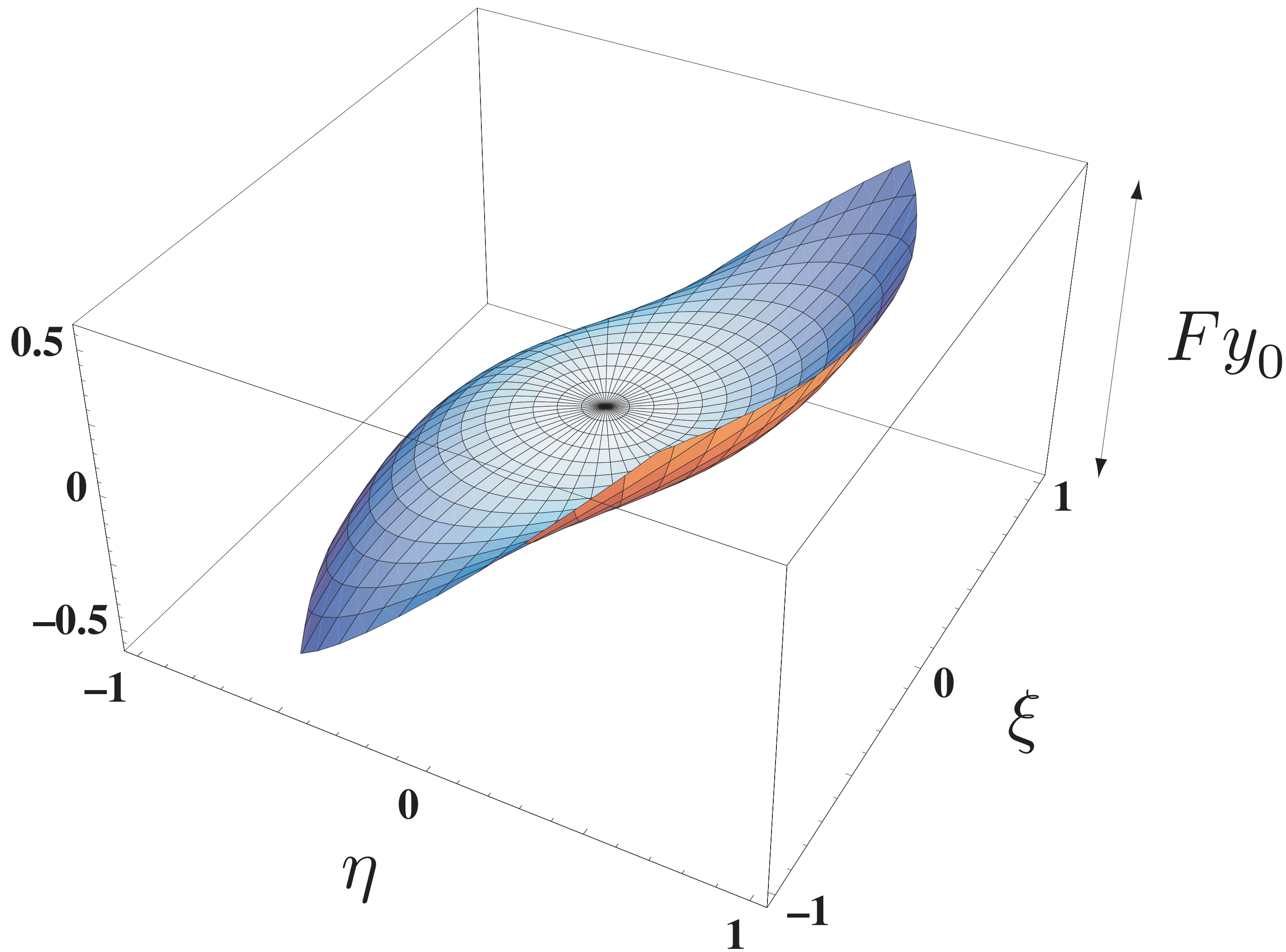
$$\phi = -\frac{1}{4}B\rho^4$$

COMA

- Coma occurs when rays that pass at different distances from the center of the entrance pupil are focused at laterally separated locations
- The Optica ray trace below illustrates positive coma (and other aberrations) of a biconvex lens
  - ▷ Lens made of BK7 glass; aperture 50 mm
  - ▷ Black dots show ray intersections with the Gaussian focal plane
  - ▷ Green dots show intersections of symmetrical pairs of marginal rays



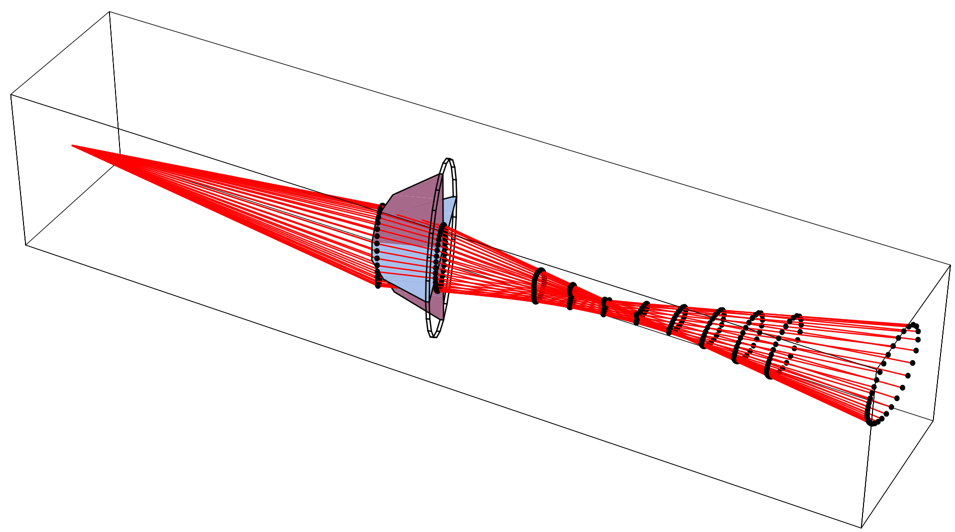
Coma

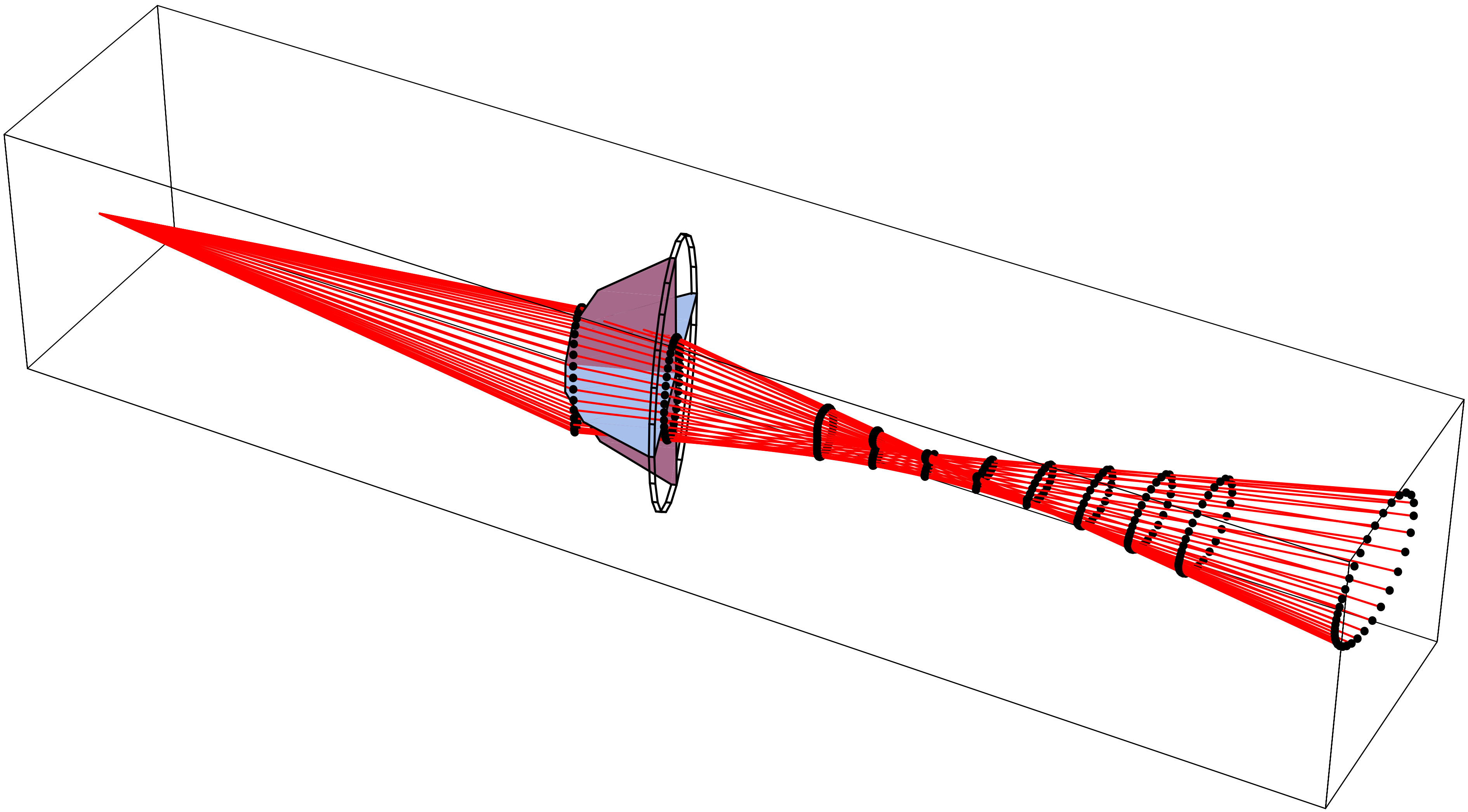


$$\phi = Fy_0\,\rho^3 \cos\theta$$

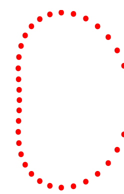
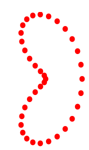
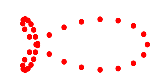
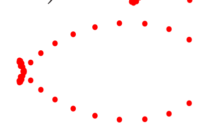
ASTIGMATISM (1)

- The **tangential plane** is the plane that contains both the optical axis and the chief ray
- The **sagittal** (or **radial**) **plane** is the plane that contains the chief ray and is perpendicular to the tangential plane
- Astigmatism occurs when the sagittal and tangential focal lengths are unequal, and a cone of rays is (approximately) focused to two separated lines
- The Optica ray trace below illustrates astigmatism (and other aberrations)

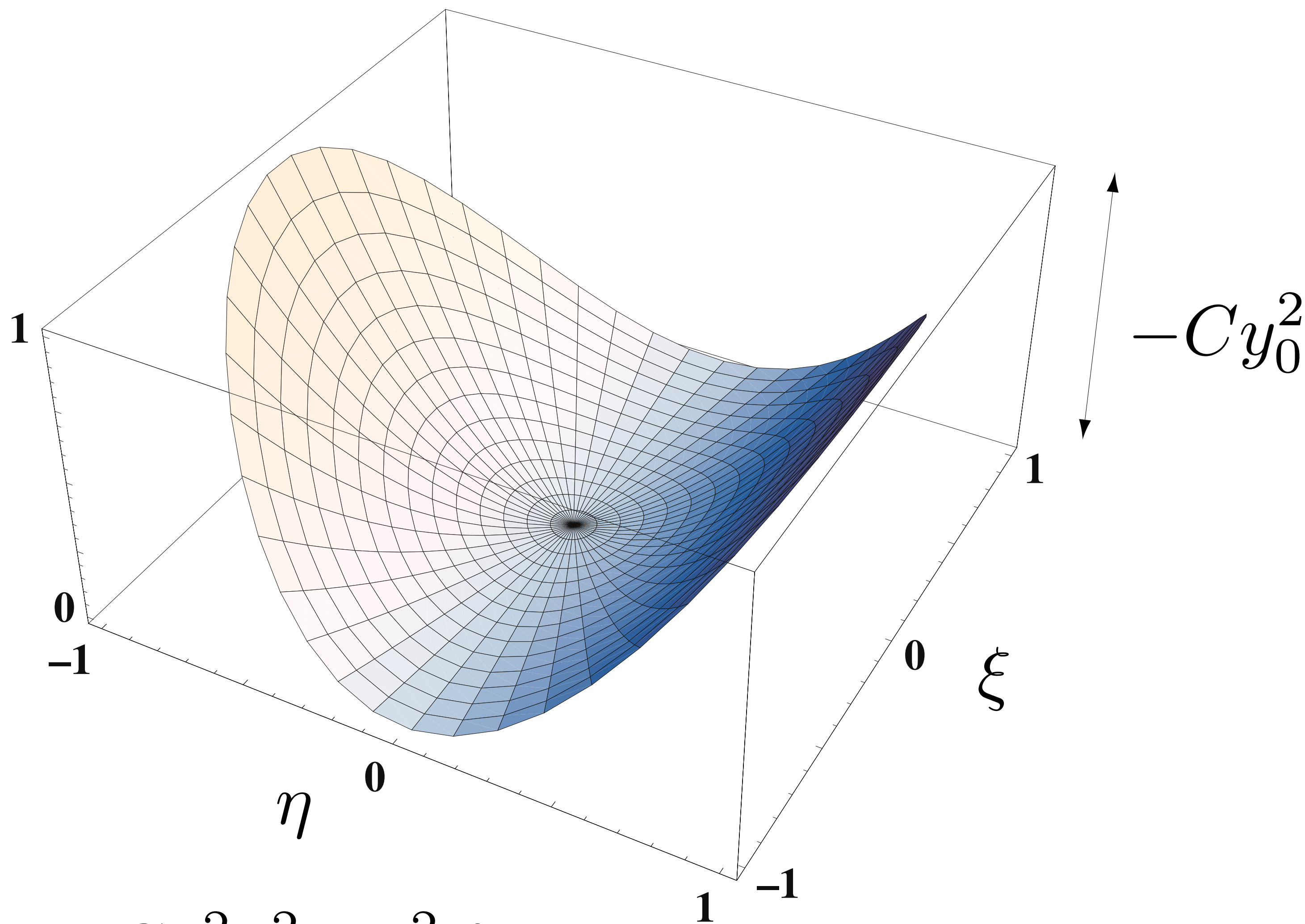




ASTIGMATISM (2)

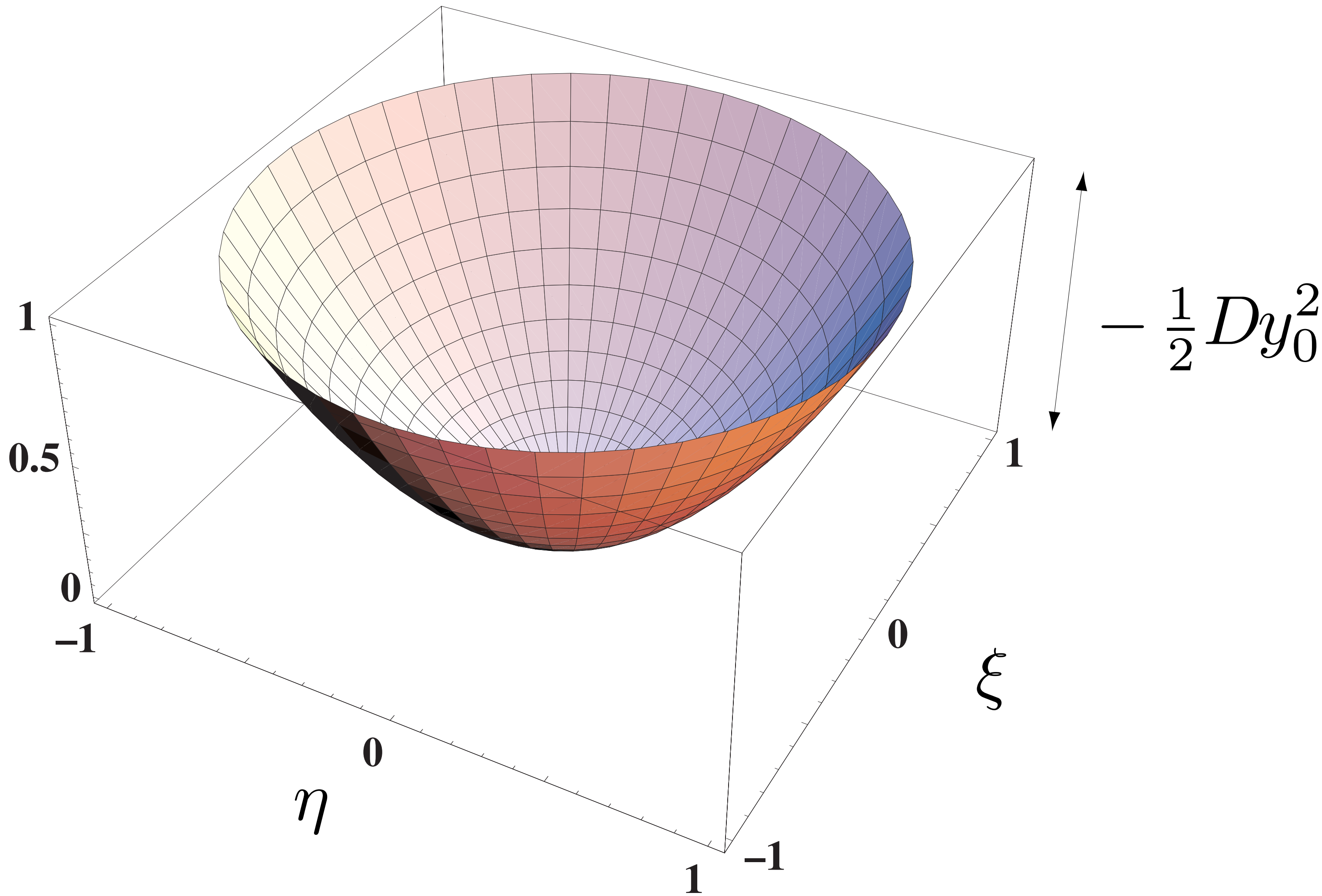
- In the preceding Optica ray trace, the tangential plane is the horizontal plane that passes through the object point
- The sagittal plane is the vertical plane that contains the object point
- Ray intersections with the planes in the figure:
  - ▷ Plane 1 (approximately elliptical converging bundle): 
  - ▷ Plane 2 (approximate line focus in the sagittal plane): 
  - ▷ Plane 3 ("circle" of least confusion): 
  - ▷ Plane 4 (approximate line focus in the tangential plane): 
  - ▷ Plane 5 (approximately elliptical diverging bundle): 
- These images show coma as well as astigmatism

Astigmatism



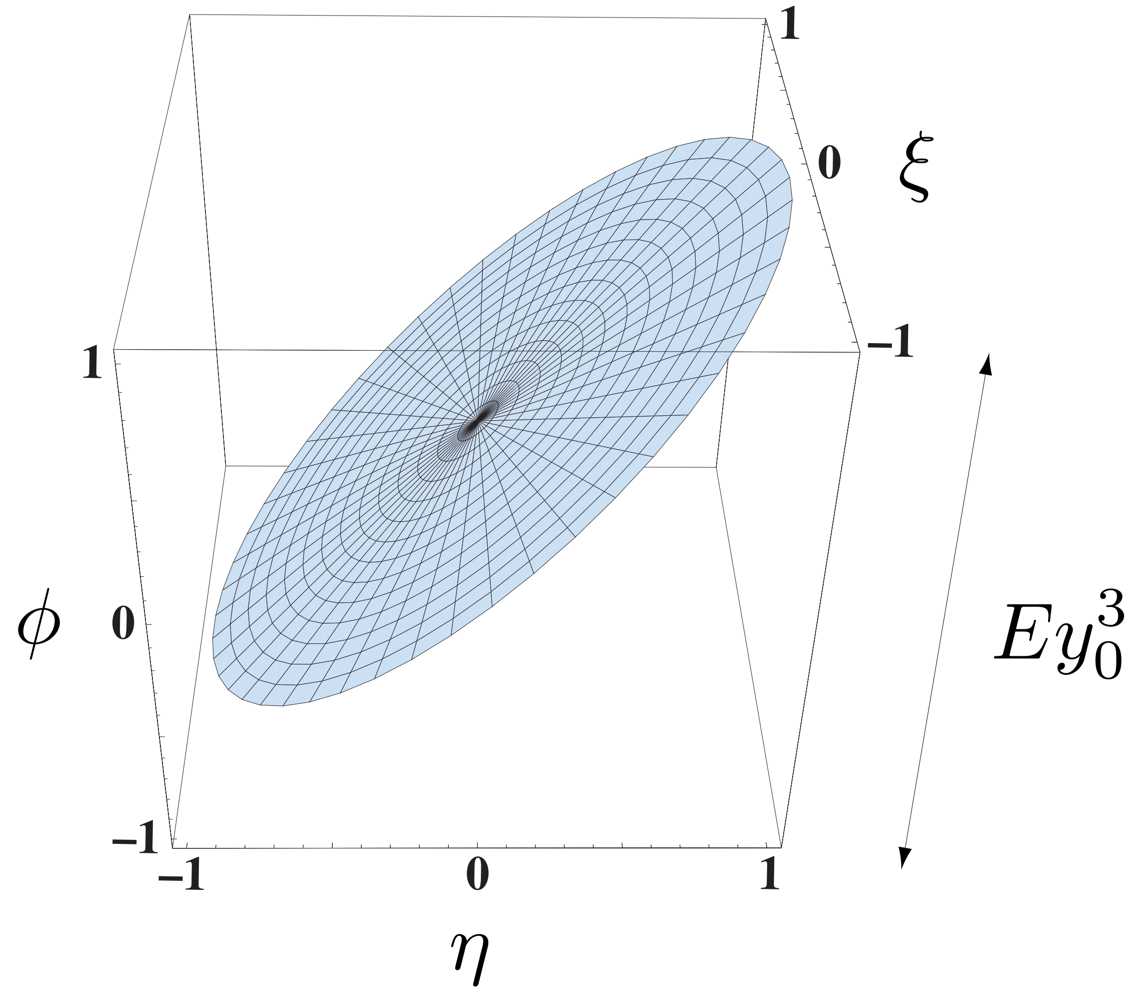
$$\phi = -Cy_0^2\rho^2\cos^2\theta$$

Curvature of Field



$$\phi = -\tfrac{1}{2}D\,y_0^2\rho^2$$

Distortion



$$\phi = Ey_0^3 \rho \cos\theta$$

REFERENCES

- Eugene Hecht, *Optics*, 4th Edition (Addison-Wesley, 2002)
- John E. Greivenkamp, *Field Guide to Geometrical Optics* (SPIE Press, 2004)
- Jack D. Gaskill, *Linear Systems, Fourier Transforms, and Optics* (Wiley, 1978)
- M. Born and E. Wolf, *Principles of Optics*, Seventh Edition (Cambridge, 1999)
- A cartoon-inspired introduction to optics: *Optics for EEs*, http://www.williamson-labs.com/optical-body.htm