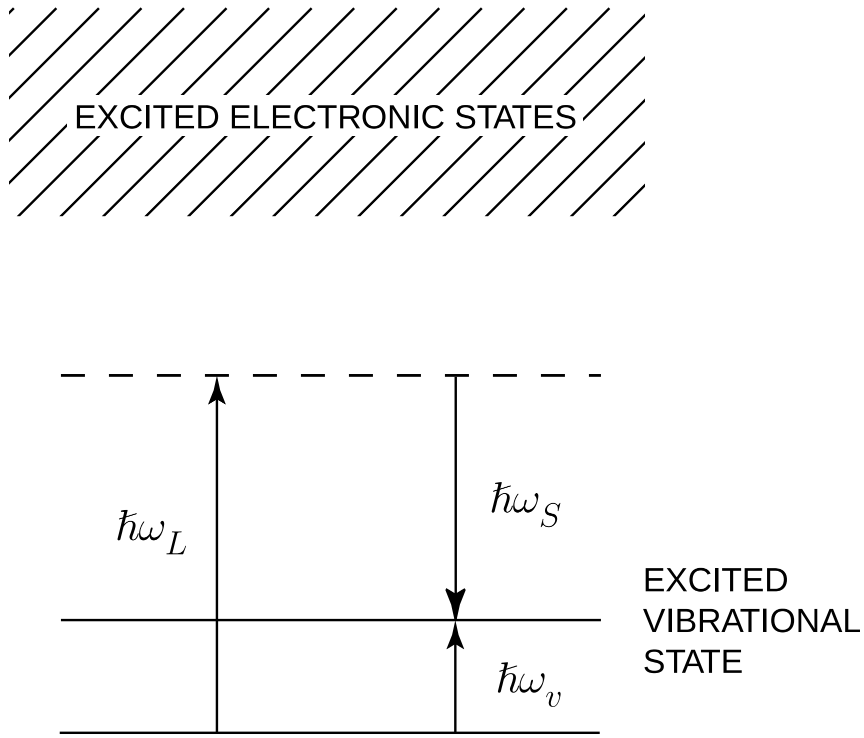
STIMULATED RAMAN SCATTERING (1)

- Origin: Inelastic scattering of light by optical phonons



STIMULATED RAMAN SCATTERING (2)

- Classical model of Raman scattering
  - ▷ Dipole moment induced in an atom or molecule by an external electric field:

$$\boldsymbol{\mu} = \alpha \mathbf{E}$$

  - ▷ The polarizability α depends on the amplitude of vibration, q:

$$\alpha = \alpha_0 + \left.\frac{\partial \alpha}{\partial q}\right|_{q=0} q + \cdots$$

  - ▷ Approximate expression for macroscopic electric polarization:

$$\mathbf{P} = N\boldsymbol{\mu} = N\alpha\mathbf{E} = N\left(\alpha_0 + \left.\frac{\partial \alpha}{\partial q}\right|_{q=0} q + \cdots\right)\mathbf{E} = \mathbf{P}_L + \mathbf{P}_{NL}$$

    - ∘ Linear polarization: $\mathbf{P}_L = N\alpha_0\mathbf{E}$ (same frequency as $\mathbf{E}$)
    - ∘ Nonlinear polarization: $\boxed{\mathbf{P}_{NL} = N\left.\frac{\partial \alpha}{\partial q}\right|_{q=0} q\mathbf{E}}$ (different frequency)

STIMULATED RAMAN SCATTERING (3)

- Work done per molecule by a slowly-turned-on external field in inducing a dipole moment:

$$\begin{aligned} dW &= \mathbf{E} \cdot d\boldsymbol{\mu} \\ &= \mathbf{E} \cdot (\alpha d\mathbf{E}) \\ &= d(\tfrac{1}{2}\alpha \mathbf{E}^2) \end{aligned}$$

$$\boxed{W = \tfrac{1}{2}\alpha \mathbf{E}^2}$$

▷ Force on a molecular vibrational coördinate q:

$$F = \frac{\partial W}{\partial q} = \frac{1}{2} \left. \frac{\partial \alpha}{\partial q} \right|_{q=0} \mathbf{E}^2$$

STIMULATED RAMAN SCATTERING (4)

- Equation of motion for vibrational coördinate:

$$m\frac{d^2q}{dt^2} + m\Gamma\frac{dq}{dt} + m\omega_v^2 q = F = \tfrac{1}{2}\left.\frac{\partial\alpha}{\partial q}\right|_{q=0} \mathbf{E}^2$$

  ▷ Assume that $\mathbf{E}$ is the sum of a Stokes wave, $\mathbf{E}_S$, and a co-propagating laser (pump) wave, $\mathbf{E}_L$:

$$\mathbf{E} = \mathbf{E}_L + \mathbf{E}_S$$

$$\mathbf{E}_L(\mathbf{r}_T, z, t) = \mathrm{Re}\left[\hat{\mathbf{e}}_L \psi_L(\mathbf{r}_T)\mathcal{E}_L(z,\, t)e^{i(\beta_L z - \omega_L t)}\right]$$

$$\mathbf{E}_S(\mathbf{r}_T, z, t) = \mathrm{Re}\left[\hat{\mathbf{e}}_S \psi_S(\mathbf{r}_T)\mathcal{E}_S(z,\, t)e^{i(\beta_S z - \omega_S t)}\right]$$

$$\begin{aligned}(\mathbf{E}_L + \mathbf{E}_S)^2 &= \mathbf{E}_L^2 + 2\mathbf{E}_L \cdot \mathbf{E}_S + \mathbf{E}_S^2 \\ &= \tfrac{1}{4}[2(\hat{\mathbf{e}}_L \cdot \hat{\mathbf{e}}_S)\psi_L(\mathbf{r}_T)\psi_S(\mathbf{r}_T)^* \\ &\qquad \times \mathcal{E}_L(z,\, t)\mathcal{E}_S(z,\, t)^* e^{i[(\beta_L - \beta_S)z - (\omega_L - \omega_S)t]} + \cdots]\end{aligned}$$

STIMULATED RAMAN SCATTERING (5)

- Interference between $\mathbf{E}_L$ and $\mathbf{E}_S$ drives the vibrational motion:

$$\frac{d^2q}{dt^2} + \Gamma\frac{dq}{dt} + \omega_v^2 q = F(t) = \frac{(\hat{\mathbf{e}}_L \cdot \hat{\mathbf{e}}_S)}{4m} \left.\frac{\partial\alpha}{\partial q}\right|_{q=0} \psi_L\psi_S^*\mathcal{E}_L\mathcal{E}_S^* e^{i[(\beta_L-\beta_S)z-(\omega_L-\omega_S)t]}$$

▷ Assume that q oscillates at $\omega_L - \omega_S$:

$$q = \frac{\hat{\mathbf{e}}_L \cdot \hat{\mathbf{e}}_S}{2}\left(\mathcal{Q}e^{i[(\beta_L-\beta_S)z-(\omega_L-\omega_S)t]}\psi_L\psi_S^* + \mathcal{Q}^* e^{-i[(\beta_L-\beta_S)z-(\omega_L-\omega_S)t]}\psi_L^*\psi_S\right)$$

Resulting equation for $\mathcal{Q}$:

$$\left[\omega_v^2 - (\omega_L-\omega_S)^2 - i(\omega_L-\omega_S)\Gamma\right]\mathcal{Q} + \left[\Gamma - 2i(\omega_L-\omega_S)\right]\frac{d\mathcal{Q}}{dt} + \frac{d^2\mathcal{Q}}{dt^2}$$

$$= \frac{1}{2m}\left.\frac{\partial\alpha}{\partial q}\right|_{q=0} \mathcal{E}_L(z,t)\mathcal{E}_S^*(z,t)$$

▷ In the steady state, $\ddot{\mathcal{Q}} = \dot{\mathcal{Q}} = 0$

STIMULATED RAMAN SCATTERING (6)

- The Green function for the vibrational motion satisfies the equation

$$\frac{d^2 g(t,t')}{dt^2} + 2\gamma \frac{dg(t,t')}{dt} + \omega_v^2 g(t,t') = \delta(t-t') \quad (\text{where} \quad \Gamma = 2\gamma)$$

and the causality condition: $\forall\, t' > t : g(t,t') = 0$

▷ Particular integral:

$$q(t) = \frac{1}{m}\int_{-\infty}^{\infty} g(t,t')F(t')dt'$$

▷ Green function:

$$g(t,t') = \frac{1}{\omega_v'} \sin\left(\omega_v'(t-t')\right) e^{-\gamma(t-t')}\theta(t-t')$$

where $\omega_v' = \sqrt{\omega_v^2 - \gamma^2}$ and θ is the unit step function

▷ Vibrational amplitude when $\omega_v' \approx \omega_v$:

$$q(t) = \frac{1}{m\omega_v'} \left.\frac{\partial\alpha}{\partial q}\right|_{q=0} \int_{-\infty}^{t} \sin\left(\omega_v'(t-t''')\right) e^{-\gamma(t-t''')} \mathbf{E}_L(t''') \cdot \mathbf{E}_R(t''')\, dt'''$$

STIMULATED RAMAN SCATTERING (7)

- Steady-state solution for Raman vibrational amplitude:

$$\mathcal{Q}(z,t) = \frac{1}{2m} \left.\frac{\partial \alpha}{\partial q}\right|_{q=0} \frac{\mathcal{E}_L(z,t)\mathcal{E}_S(z,t)^*}{\omega_v^2 - (\omega_L - \omega_S)^2 - i(\omega_L - \omega_S)\Gamma}$$

▷ When $\Gamma \ll \omega_v$, the denominator is approximately Lorentzian:

$$\begin{aligned} \omega_v^2 - (\omega_L - \omega_S)^2 - i(\omega_L - \omega_S)\Gamma &\approx (\omega_v - \omega_L + \omega_S)(\omega_v + \omega_L - \omega_S) - i\Gamma\omega_v \\ &\approx 2\omega_v \left[\omega_v - \omega_L + \omega_S - i\frac{\Gamma}{2}\right] \end{aligned}$$

STIMULATED RAMAN SCATTERING (8)

- Steady-state approximation to $\mathcal{Q}$:

$$\mathcal{Q}(z,t) \approx \frac{1}{4m\omega_v} \left.\frac{\partial \alpha}{\partial q}\right|_{q=0} \frac{\mathcal{E}_L(z,t)\mathcal{E}_S(z,t)^*(\omega_v - \omega_L + \omega_S + i\Gamma/2)}{(\omega_v - \omega_L + \omega_S)^2 + (\Gamma/2)^2}$$

- Vibrational envelope when $\omega_v = \omega_L - \omega_S$ (on the Raman resonance):

$$\mathcal{Q}(z,t) = \frac{i}{2m\Gamma\omega_v} \left.\frac{\partial \alpha}{\partial q}\right|_{q=0} \mathcal{E}_L(z,t)\mathcal{E}_S^*(z,t)$$

STIMULATED RAMAN SCATTERING (9)

- Slowly varying contribution, near the Stokes frequency, ω_s, and in the direction of the Stokes electric field, $\hat{\mathbf{e}}_S$, to the Raman electric polarization $\mathbf{P}_{NL} = N \left.\frac{\partial\alpha}{\partial q}\right|_{q=0} q\mathbf{E}$:

$$\mathbf{P}_{NL} = \frac{-i}{2}\left(\mathcal{P}_S\psi_S e^{-i\omega_S t'}\hat{\mathbf{e}}_S + \mathcal{P}_S^*\psi_S^* e^{i\omega_S t'}\hat{\mathbf{e}}_S\right) + \text{terms at other frequencies}$$

$$\begin{aligned} q\mathbf{E} &= \frac{\hat{\mathbf{e}}_L \cdot \hat{\mathbf{e}}_S}{4}\left(\mathcal{Q}\psi_L\psi_S^* e^{-i(\omega_L-\omega_S)t'} + \mathcal{Q}^*\psi_L^*\psi_S e^{i(\omega_L-\omega_S)t'}\right) \\ &\quad \times \left(\mathcal{E}_L\psi_L e^{-i\omega_L t'}\hat{\mathbf{e}}_L + \mathcal{E}_L^*\psi_L^* e^{i\omega_L t'}\hat{\mathbf{e}}_L + \cdots\right) \\ &= \frac{\hat{\mathbf{e}}_L \cdot \hat{\mathbf{e}}_S}{4}\mathcal{Q}^*\mathcal{E}_L|\psi_L|^2\psi_S\,\hat{\mathbf{e}}_L e^{-i\omega_S t'} + \text{terms at other frequencies} \end{aligned}$$

Then

$$\mathcal{P}_S(z,t) = \frac{i}{2}N\left.\frac{\partial\alpha}{\partial q}\right|_{q=0}\mathcal{Q}^*(z,t)\mathcal{E}_L(z,t)(\hat{\mathbf{e}}_L \cdot \hat{\mathbf{e}}_S)^2\frac{\langle\psi_S, |\psi_L|^2\psi_S\rangle}{\langle\psi_S, \psi_S\rangle}$$

Note that $\mathcal{P}_S$ is proportional to the product $\mathcal{Q}^*\mathcal{E}_L$

STIMULATED RAMAN SCATTERING (10)

- Contribution at the Stokes frequency ω_s to the Raman electric polarization $\mathbf{P}_{NL}(\mathbf{r}_T, z, t) = N \left.\frac{\partial\alpha}{\partial q}\right|_{q=0} q(\mathbf{r}_T, z, t)\mathbf{E}(\mathbf{r}_T, z, t)$ (including mode functions):

$$\mathcal{P}_S(z,t) = \frac{i}{2} N \left.\frac{\partial\alpha}{\partial q}\right|_{q=0} \frac{\iint |\psi_L|^2 |\psi_S|^2 \, d^2 r_T}{\iint |\psi_S|^2 \, d^2 r_T} (\hat{\mathbf{e}}_L \cdot \hat{\mathbf{e}}_S)^2 \, \mathcal{E}_L(z,t)\mathcal{Q}^*(z,t)$$

▷ Fiber paraxial wave equation for Raman-Stokes scattering:

$$\frac{\partial\overline{\mathcal{E}}_S}{\partial z'} = -\frac{\alpha}{2}\overline{\mathcal{E}}_S + \frac{2\pi\omega_S}{n_0 c}\overline{\mathcal{P}}_S$$

STIMULATED RAMAN SCATTERING (11)

- Fiber paraxial wave equation for Raman-Stokes scattering:

$$\frac{\partial \overline{\mathcal{E}}_S}{\partial z'} = -\frac{\alpha}{2}\overline{\mathcal{E}}_S + \tfrac{1}{2} g_R(\Delta\omega) \left|\overline{\mathcal{E}}_L\right|^2 \overline{\mathcal{E}}_S$$

▷ Detuning from peak of Raman gain curve:

$$\Delta\omega = \omega_v - \omega_L + \omega_S$$

▷ Frequency-dependent Raman gain:

$$g_R(\Delta\omega) = g_R(0) \frac{(\Gamma/2)^2}{(\Delta\omega)^2 + (\Gamma/2)^2} \left(\hat{\mathbf{e}}_L \cdot \hat{\mathbf{e}}_S\right)^2$$

▷ Maximum Raman gain:

$$\boxed{g_R(0) = \frac{\pi N \omega_S}{c n_0 m \omega_v \Gamma} \left(\left.\frac{\partial \alpha}{\partial q}\right|_{q=0} \right)^2}$$

▷ W. Kaiser and M. Maier, "Stimulated Rayleigh, Raman and Brillouin Spectroscopy", in F. T. Arecchi and E. O. Sculz-Dubois, editors, *Laser Handbook*, Vol. 2, pp. 1077–1150, Eq. (30).

PROPAGATION OF RAMAN-STOKES POWER

- Fiber paraxial wave equation for Raman-Stokes scattering:

$$\frac{\partial \overline{\mathcal{E}}_S}{\partial z'} = -\frac{\alpha}{2}\overline{\mathcal{E}}_S + \tfrac{1}{2} g_R(\Delta\omega)\, |\overline{\mathcal{E}}_L|^2\, \overline{\mathcal{E}}_S$$

▷ Multiply $\partial \overline{\mathcal{E}}_S / \partial z'$ by $\overline{\mathcal{E}}_S^{\,*}$, take the complex conjugate, and add:

$$\frac{\partial \overline{\mathcal{E}}_S}{\partial z'}\, \overline{\mathcal{E}}_S^{\,*} + \overline{\mathcal{E}}_S\, \frac{\partial \overline{\mathcal{E}}_S^{\,*}}{\partial z'} = \frac{\partial}{\partial z'} |\overline{\mathcal{E}}_S|^2$$

▷ Propagation equation for the intensity:

$$\frac{\partial}{\partial z'} |\overline{\mathcal{E}}_S|^2 = -\alpha |\overline{\mathcal{E}}_S|^2 + g_R(\Delta\omega)\, |\overline{\mathcal{E}}_L|^2 |\overline{\mathcal{E}}_S|^2$$

▷ Normalize: $\overline{\mathcal{F}}_L = (c n_{0,L} A_e / 8\pi)^{1/2}\, \overline{\mathcal{E}}_L \;\Rightarrow\;$ laser power is $P_L = |\overline{\mathcal{F}}_L|^2$

$$\boxed{\frac{\partial}{\partial z'} P_S = -\alpha P_S + \left(\frac{8\pi g_R(\Delta\omega)}{c n_{0,L} A_e} \right) P_L P_S}$$

STIMULATED RAMAN SCATTERING (12)

- Raman gain:

$$g_R(0) = \frac{\pi N \omega_S}{c n_0 m \omega_v \Gamma} \left(\left. \frac{\partial \alpha}{\partial q} \right|_{q=0} \right)^2$$

▷ The peak Raman gain is proportional to the Stokes frequency, ω_S

▷ Propagation equation for Raman-Stokes power in practical units:

$$\frac{dP_S}{dz'} = -\alpha P_S + (g'_R P_L) \frac{P_S}{A_e} \qquad \text{where} \qquad g'_R(0) = \frac{8\pi}{c n_0} g_R(0)$$

▷ In SiO_2 fiber:

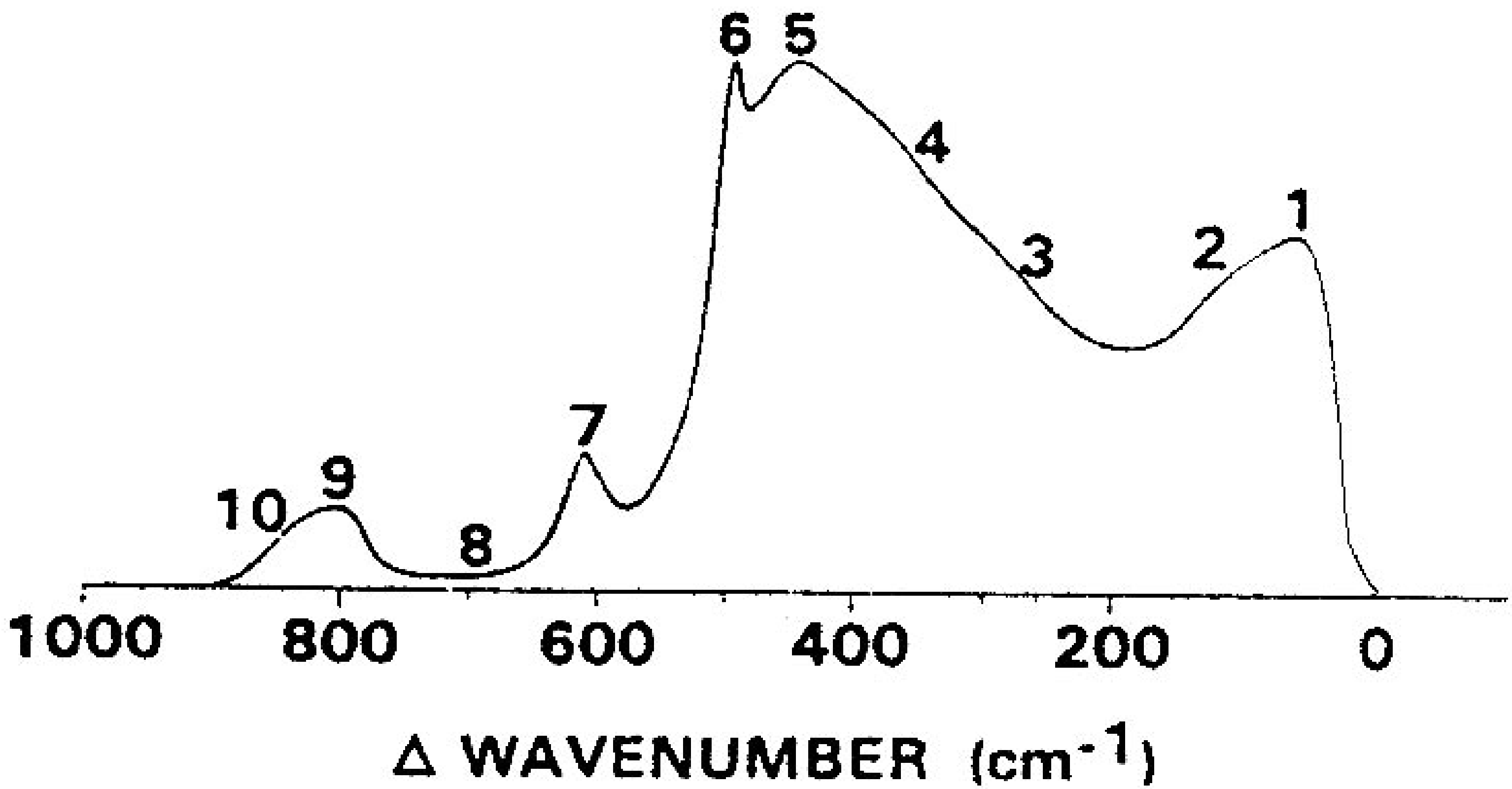
$$g'_R(0) \approx \frac{1 \times 10^{-22}}{\lambda_S\ (\text{cm})} \frac{\text{cm} \cdot \text{s}}{\text{erg}} = \frac{1 \times 10^{-11}}{\lambda_S\ (\mu\text{m})}\ \text{cm/W}, \qquad \omega_v \approx 12\text{–}15 \times 10^3\ \text{GHz}$$

▷ Crude fit to observed frequency dependence:

$$g'_R(\Delta\omega) \approx \begin{cases} \dfrac{(\nu_L - \nu_S)\ (\text{Hz})}{1.5 \times 10^{13}} g'_R(0), & \text{if } \nu_L - \nu_S < 1.5 \times 10^{13}\ \text{Hz}; \\ 0, & \text{otherwise.} \end{cases}$$

RAMAN GAIN SPECTRUM OF SILICA FIBER

- From G.E.Walrafen and P.N.Krishnan, "Model analysis of the Raman spectrum from fused silica optical fibers", *Applied Optics* **21**, 359-360 (1982)



UNITS FOR SRS GAIN

- Basic cgs mechanical units:

$$[\text{work}] = \frac{\text{gm}\cdot\text{cm}^2}{\text{s}^2} \qquad \text{unit:}\quad 1\ \text{erg} = 10^{-7}\text{Joule}$$

- Basic Gaussian cgs electromagnetic units:

$$[\text{charge}] = \left(\frac{\text{gm}\cdot\text{cm}^3}{\text{s}^2}\right)^{1/2} \qquad \text{unit:}\quad \text{stat-Coulomb (stC)}$$

$$[\mathbf{E}] = [\mathbf{P}] = \left(\frac{\text{gm}}{\text{cm}\cdot\text{s}^2}\right)^{1/2} = \left(\frac{\text{erg}}{\text{cm}^3}\right)^{1/2} \qquad \text{unit:}\quad \text{stat-Volt per cm (stV/cm)}$$

$$[\alpha] = \text{cm}^3, \qquad \left[\left.\frac{\partial\alpha}{\partial q}\right|_{q=0}\right] = \text{cm}^2, \qquad [N] = \text{cm}^{-3}$$

- Units of Raman gain:

$$[g_R] = \frac{\text{s}^2}{\text{gm}} = \frac{\text{cm}^2}{\text{erg}}, \qquad [g_R'] = \frac{\text{cm}\cdot\text{s}}{\text{erg}}$$

LENGTH SCALING FOR SRS

- Normalized fiber paraxial wave equation for Raman-Stokes scattering, assuming a **co-propagating** laser (pump) wave:

$$\frac{\partial}{\partial z'} P_S(z') = -\alpha P_S(z') + \frac{1}{L_R} \frac{P_L(z')}{P_L(0)} P_S(z')$$

▷ Characteristic length for e-fold growth of the Raman-Stokes wave:

$$L_R = \frac{c n_{0,L} A_e}{8\pi f_R g_R(\Delta\omega) P_L(0)} = \frac{A_e}{f_R g'_R(\Delta\omega) P_L(0)} \approx 3750\,\mathrm{km} \text{ if } P_L(0) = 1\,\mathrm{mW}$$

- Normalized fiber paraxial wave equation for Raman-Stokes scattering, assuming a **counter-propagating** laser (pump) wave:

$$\frac{\partial}{\partial z'} P_S(z') = -\alpha P_S(z') + \frac{1}{L_R} \frac{P_L(z')}{P_L(L)} P_S(z')$$

▷ Characteristic length for e-fold growth of the Raman-Stokes wave:

$$L_R = \frac{c n_{0,L} A_e}{8\pi f_R g_R(\Delta\omega) P_L(L)} \approx 3.75 \text{ km if } P_L(L) = 1 \text{ W}$$

PROPAGATION OF SRS POWER IN TERMS OF POLARIZATION

- Fiber paraxial wave equation for Raman-Stokes scattering:

$$\frac{\partial \overline{\mathcal{E}}_S}{\partial z'} = -\frac{\alpha}{2}\overline{\mathcal{E}}_S + \frac{2\pi\omega_S}{n_0 c}\overline{\mathcal{P}}_S$$

  - ▷ Multiply $\partial\overline{\mathcal{E}}_S/\partial z'$ by $\overline{\mathcal{E}}_S^{\,*}$, take the complex conjugate, and add to get $(\partial/\partial z')|\overline{\mathcal{E}}_S|^2$
  - ▷ Propagation equation for the intensity:

$$\frac{\partial}{\partial z'}|\overline{\mathcal{E}}_S|^2 = -\alpha|\overline{\mathcal{E}}_S|^2 + \frac{4\pi\omega_S}{n_0 c}\,\mathrm{Re}\left(\overline{\mathcal{P}}_S^{\,*}\overline{\mathcal{E}}_S\right)$$

  - ▷ Normalize: $\overline{\mathcal{F}}_L = (cn_{0,L}A_e/8\pi)^{1/2}\,\overline{\mathcal{E}}_L \;\Rightarrow\;$ laser power is $P_L = |\overline{\mathcal{F}}_L|^2$

$$\boxed{\frac{\partial}{\partial z'}P_S = -\alpha P_S + \frac{\omega_S A_e}{2}\,\mathrm{Re}\left(\overline{\mathcal{P}}_S^{\,*}\overline{\mathcal{E}}_S\right)}$$

STIMULATED RAMAN SCATTERING (13)

- Power amplification by stimulated Raman scattering:
  - ▷ From the gain equation:

$$P_S(L) = e^{g'_R(0) P_L L_e(L)/A_{\text{eff}} - \alpha L} P_S(0)$$

    - ∘ This assumes that the laser pump is not depleted
  - ▷ From the paraxial wave equation:
    - ∘ Growth of Stokes power:

$$\frac{dP_S}{dz} = -\alpha P_S + \omega_S \left(\frac{g'_R(0)}{\omega_S} \right) \frac{P_L}{A_{\text{eff}}} P_S$$

    - ∘ Depletion of laser pump power:

$$\frac{dP_L}{dz} = -\alpha P_L - \omega_L \left(\frac{g'_R(0)}{\omega_S} \right) \frac{P_S}{A_{\text{eff}}} P_L$$

STIMULATED RAMAN SCATTERING (14)

- The Manley-Rowe relations for SRS:
  - ▷ Photon fluxes:

$$N_S = \frac{P_S}{\hbar\omega_S}, \qquad N_L = \frac{P_L}{\hbar\omega_L}$$

  - ▷ At zero attenuation ($\alpha = 0$),

$$\frac{1}{\hbar\omega_S}\frac{dP_S}{dz} = \frac{dN_S}{dz} = \frac{\hbar\omega_L g_R'(0)}{A_{\text{eff}}} N_L N_S$$

$$\frac{1}{\hbar\omega_L}\frac{dP_L}{dz} = \frac{dN_L}{dz} = -\frac{\hbar\omega_L g_R'(0)}{A_{\text{eff}}} N_L N_S$$

  - ▷ The **Manley-Rowe relations**:

$$\boxed{\frac{dN_L}{dz} = -\frac{dN_S}{dz}}$$

SPONTANEOUS vs. STIMULATED RAMAN SCATTERING

- Propagation equation for Raman-Stokes photon number $N_S \propto P_S$:

$$\frac{dN_S}{dz} = -\alpha_S N_S(z) + g'_R(0) I_L(z) \Bigg(\underbrace{N_S(z)}_{\text{stimulated}} + \underbrace{1}_{\text{spontaneous}} \Bigg)$$

▷ **Spontaneous** scattering limit ($N_S \ll 1$):

$$N_S(z) \approx \int_0^L g'_R(0) I_L(z)\, dz = g'_R(0) I_L(0) L_{\text{eff}}(L)$$

Stokes photon number flux grows **linearly** when $z \ll \alpha^{-1}$

▷ **Stimulated** scattering limit ($N_S \gg 1$):

$$N_S(z) \approx e^{g'_R(0) I_L(0) L_{\text{eff}}(L) - \alpha L}$$

Stokes photon number flux grows **exponentially** when $z \ll \alpha^{-1}$

STIMULATED RAMAN SCATTERING (15)

- Limits on number N of WDM channels imposed by SRS:
  - ▷ Reference: A. R. Chraplyvy, *Electronics Letters* **20**, 58–59 (1984)
  - ▷ The highest-frequency channel ($n = 0$) is depleted by Raman-Stokes scattering into the $N - 1$ other channels, using a crude spectral fit:

$$\frac{1}{P_0}\frac{dP_0}{dz} = -\alpha - \frac{P}{A_{\text{eff}}}\sum_{n=1}^{N-1}\frac{\omega_n}{\omega_0}g_R'(0)\frac{n\Delta\nu}{1.5\times 10^{13}}$$
$$= -\alpha - g_R'(0)\frac{P}{A_{\text{eff}}}\left(\frac{\Delta\nu}{1.5\times 10^{13}}\right)\frac{N(N-1)}{2}$$

  - ▷ Nonlinear attenuation of channel 0 by SRS:

$$\alpha L + \ln\frac{P_0(L)}{P_0(0)} = -g_R'(0)\frac{P}{A_{\text{eff}}}\left(\frac{\Delta\nu}{1.5\times 10^{13}}\right)\frac{N(N-1)}{2}L_e(L)$$

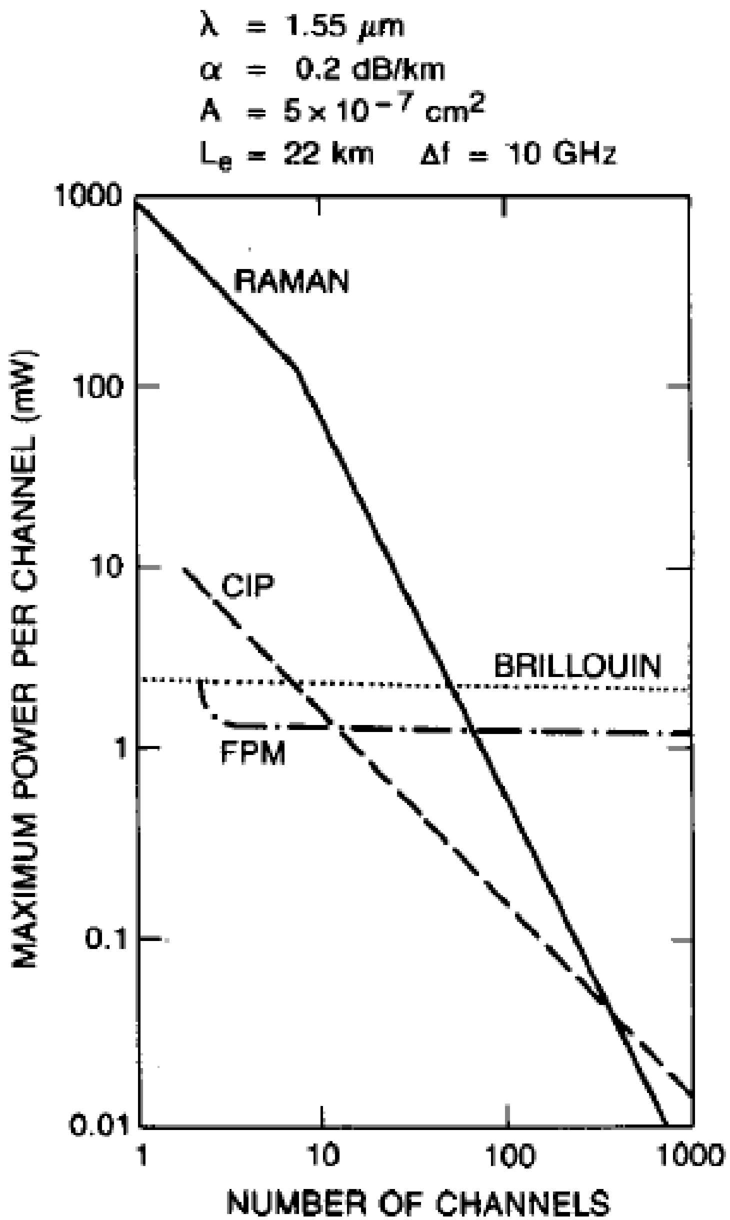


Fig. 10. Maximum power per channel versus number of channels which ensures SRS, CIP, SBS, and FPM degradations below 1 dB for all channels.

STIMULATED RAMAN SCATTERING (16)

- Spacetime approach to Raman scattering:
  - ▷ Coupled-wave theory breaks down for pulses shorter than ≈ 100 fs
  - ▷ Approach of Blow & Wood: Account for Raman scattering through a time-delayed response function, replacing $\overline{\mathfrak{F}}(z', t')|\overline{\mathfrak{F}}(z', t')|^2$ with

$$\overline{\mathfrak{F}}(z', t') \int_{-\infty}^{\infty} r(t'-t'')|\overline{\mathfrak{F}}(z', t'')|^2 dt'' = \overline{\mathfrak{F}}(z', t') \int_{-\infty}^{\infty} r(t'')|\overline{\mathfrak{F}}(z', t'-t'')|^2 dt''$$

  - ▷ Electronic (instantaneous) + vibrational (delayed) response:

$$r(t) = (1 - f_R)\delta(t) + f_R h_R(t)$$

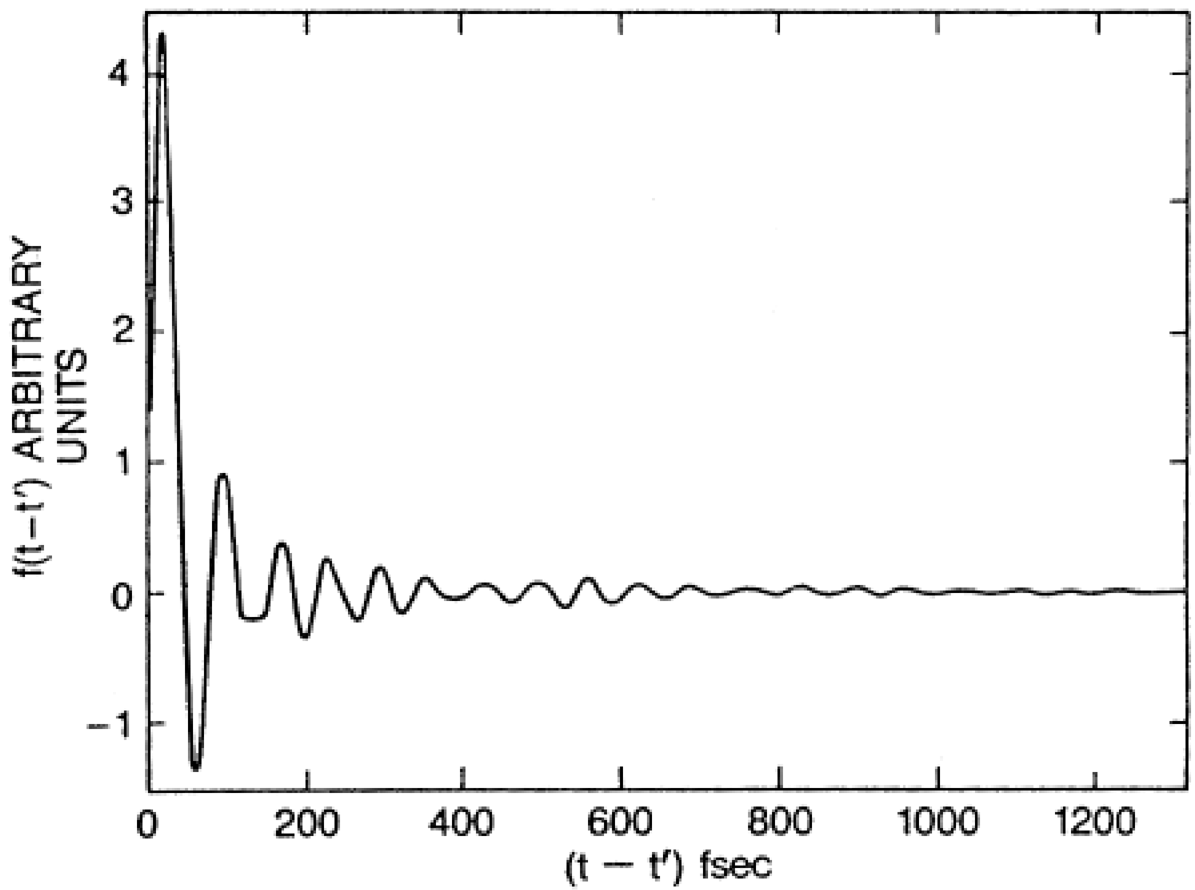
  - ▷ Delay-differential propagation equation:

$$\left[\frac{\partial}{\partial z'} + \left(i\frac{\beta_2}{2}\frac{\partial^2}{\partial t'^2} - \frac{\beta_3}{6}\frac{\partial^3}{\partial t'^3}\right)\right]\overline{\mathfrak{F}}(z', t')$$

$$= -\frac{\alpha}{2}\overline{\mathfrak{F}} + i\gamma\left(1 + i\frac{2}{\omega_0}\frac{\partial}{\partial t'}\right)\overline{\mathfrak{F}}(z', t') \int_{-\infty}^{\infty} r(t'')|\overline{\mathfrak{F}}(z', t' - t'')|^2 dt''$$

RAMAN RESPONSE FUNCTION

- Response function, $h_R(t)$, obtained from the experimental Raman gain spectrum by R. H. Stolen et al., "Raman response function of silica-core fibers", *J. Opt. Soc. Am. B* **6**, 1159-1166 (1989)



STIMULATED RAMAN SCATTERING (17)

- Method of moments applied to Raman scattering:
  - ▷ Time-delayed response

$$|\overline{\mathfrak{F}}(z',t'-t'')|^2 = |\overline{\mathfrak{F}}(z',t')|^2 - t''\frac{\partial}{\partial t'}|\overline{\mathfrak{F}}(z',t')|^2 + \cdots$$

$$\int_{-\infty}^{\infty} h_R(t'')|\overline{\mathfrak{F}}(z',t'-t'')|^2 dt'' = |\overline{\mathfrak{F}}(z',t')|^2 \int_{-\infty}^{\infty} h_R(t'')dt'' - \frac{\partial}{\partial t'}|\overline{\mathfrak{F}}(z',t')|^2 \int_{-\infty}^{\infty} t'' h_R(t'')dt''$$

  - ▷ From $\int_{-\infty}^{\infty} r(t)\,dt = 1$, get

$$\int_{-\infty}^{\infty} h_R(t)\,dt = 1$$

  - ▷ Raman response time:

$$T_R = \int_{-\infty}^{\infty} t\,h_R(t)\,dt$$

STIMULATED RAMAN SCATTERING (18)

- Generalized nonlinear Schrödinger equation including Raman scattering:

$$\left[\frac{\partial}{\partial z'} + \left(i\frac{\beta_2}{2}\frac{\partial^2}{\partial t'^2} - \frac{\beta_3}{6}\frac{\partial^3}{\partial t'^3}\right)\right]\overline{\mathfrak{F}}(z', t')$$
$$= -\frac{\alpha}{2}\overline{\mathfrak{F}} + i\gamma\left(1 + i\frac{2}{\omega_0}\frac{\partial}{\partial t'}\right)\left[(1 - 2f_r)|\overline{\mathfrak{F}}|^2\overline{\mathfrak{F}} - T_R\overline{\mathfrak{F}}\frac{\partial}{\partial t'}|\overline{\mathfrak{F}}|^2\right]$$

  ▷ Must be solved numerically

NUMERICAL METHODS (1)

- Numerical methods for the generalized nonlinear Schrödinger equation

$$\left[\frac{\partial}{\partial z'} + \left(i\frac{\beta_2}{2}\frac{\partial^2}{\partial t'^2} - \frac{\beta_3}{6}\frac{\partial^3}{\partial t'^3}\right)\right]\overline{\mathfrak{F}}(z',t')$$
$$= -\frac{\alpha}{2}\overline{\mathfrak{F}} + i\gamma\left[\left(1 + i\frac{2}{\omega_0}\frac{\partial}{\partial t'}\right)|\overline{\mathfrak{F}}|^2\overline{\mathfrak{F}} + f_R T_R \overline{\mathfrak{F}}\frac{\partial}{\partial t'}|\overline{\mathfrak{F}}|^2\right]$$

  ▷ Strategy: Discretize the time derivatives, then solve the resulting ODE in z'

  - ∘ Pseudospectral methods: Evaluate time derivatives in Fourier space, where $\partial/\partial t' \to -i\omega$
  - ∘ Finite-difference methods: Approximate time derivatives with difference quotients

NUMERICAL METHODS (2)

- Generalized nonlinear Schrödinger equation:

$$\frac{\partial \overline{\mathfrak{F}}}{\partial z'} = (\hat{D} + \hat{N})\overline{\mathfrak{F}}$$

  - ▷ Dispersion and attenuation operator:

$$\hat{D} = -\frac{\alpha}{2} - \left(i\frac{\beta_2}{2}\frac{\partial^2}{\partial t'^2} - \frac{\beta_3}{6}\frac{\partial^3}{\partial t'^3} \right)$$

    - ∘ "Diagonal" in Fourier space

  - ▷ Nonlinear operator:

$$\hat{N} = i\gamma \left[\left(1 + i\frac{2}{\omega_0}\frac{\partial}{\partial t'} \right) |\overline{\mathfrak{F}}|^2 + f_R T_R \left(\frac{\partial}{\partial t'} |\overline{\mathfrak{F}}|^2 \right) \right]$$

    - ∘ "Diagonal" neither in Fourier space nor in t'-space

NUMERICAL METHODS (3)

- Split-step Fourier method:

  $\mathfrak{F}(z'+h,t')$

  $$= F^{-1}\exp\left(\frac{h}{2}\hat{D}\right) F \exp\left(\int_{z'}^{z'+h} \hat{N}(z'')\,dz''\right) F^{-1}\exp\left(\frac{h}{2}\hat{D}\right) F\,\mathfrak{F}(z',t')$$

  ▷ F = discrete Fourier transform operator with respect to t'

  ▷ This formulation is appropriate if a single-step finite-difference method is used to approximate $\exp\left(\int_{z'}^{z'+h} \hat{N}(z'')\,dz''\right)$

  ▷ How to apply the split-step method is ambiguous if one uses a multistep method, due to the choice of whether the function for the previous step is evaluated in signal space or transform space

  - To eliminate this ambiguity, the function should be evaluated using a previous step that has been propagated in both domains

MOTIVATION FOR USING RAMAN AMPLIFICATION

- Long-distance transmission systems require amplification, preferably optical, to compensate for linear attenuation
  - ▷ Erbium-doped fiber amplifiers (EDFAs)
    - ◦ The majority of installed fiber is in the 1310 nm region. In order to use EDFAs, replacement of the installed fiber would be required due to the small amplification region centered about 1550 nm.
    - ◦ EDFAs must be placed periodically along the fiber to achieve proper amplification
  - ▷ Stimulated Raman scattering (SRS) amplification
    - ◦ Since the Raman gain spectrum can include the 1310 nm region, it is possible to use currently installed fiber
    - ◦ Useful SRS amplification can be achieved with only the addition of a counter-propagating pump beam originating at the receiver
    - ◦ However, Raman amplification may lead to spatial hole burning

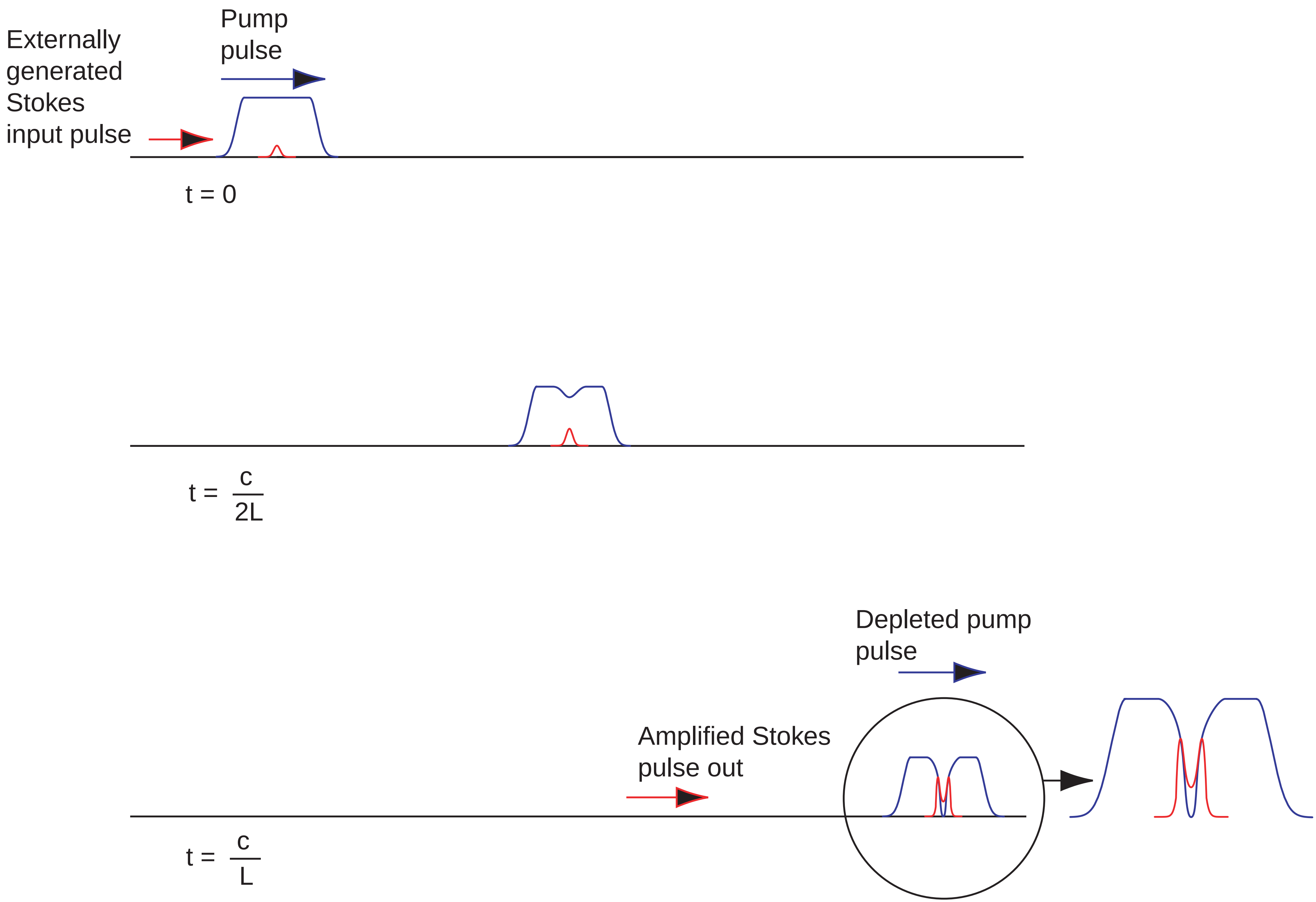
RAMAN AMPLIFICATION

- "A photon for a photon"
  - ▷ Manley-Rowe relation and Raman rate equation (for zero attenuation):

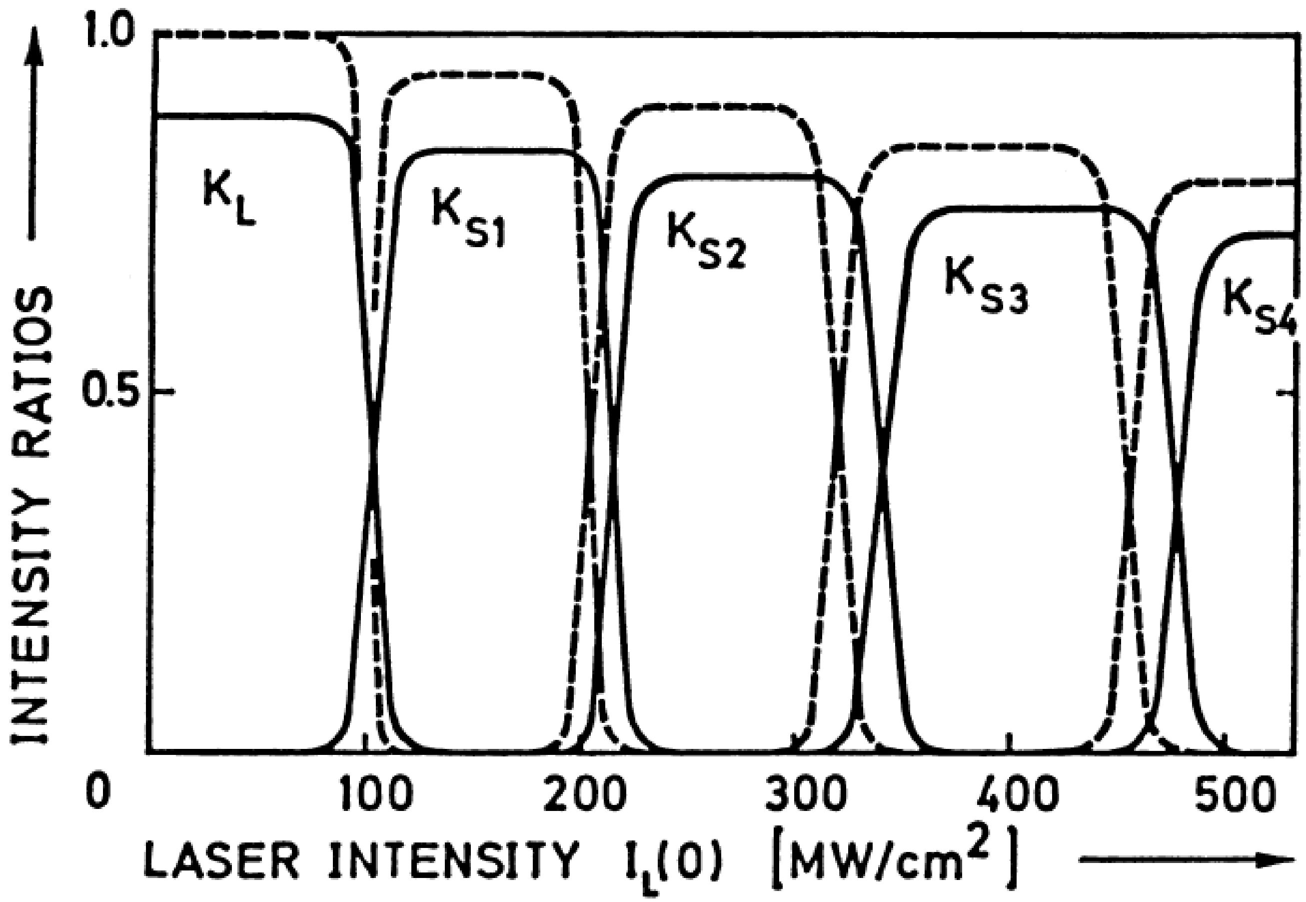
$$\frac{dN_L}{dz'} = -\frac{dN_S}{dz'} = -\frac{1}{L_R}\frac{N_L(z')}{N_L(0)}N_S(z')$$

    - ◦ $N_S = \frac{P_S}{\hbar\omega_S}$ is the Stokes photon flux; $N_L = \frac{P_L}{\hbar\omega_L}$ is the laser photon flux
- Co-propagating laser and Stokes pulses
  - ▷ 1st Stokes gain goes to zero when pump is depleted; then 1st Stokes pumps 2nd Stokes, etc.
  - ▷ 1st Stokes intensity goes to zero when pump is depleted (InSb)
- Counter-propagating laser wave and Stokes pulse
  - ▷ Gain stays constant while Stokes grows exponentially
  - ▷ Stokes wave may be many times more intense than the pump
  - ▷ Stokes pulse acquires characteristic "shark fin" shape and shifts towards earlier times

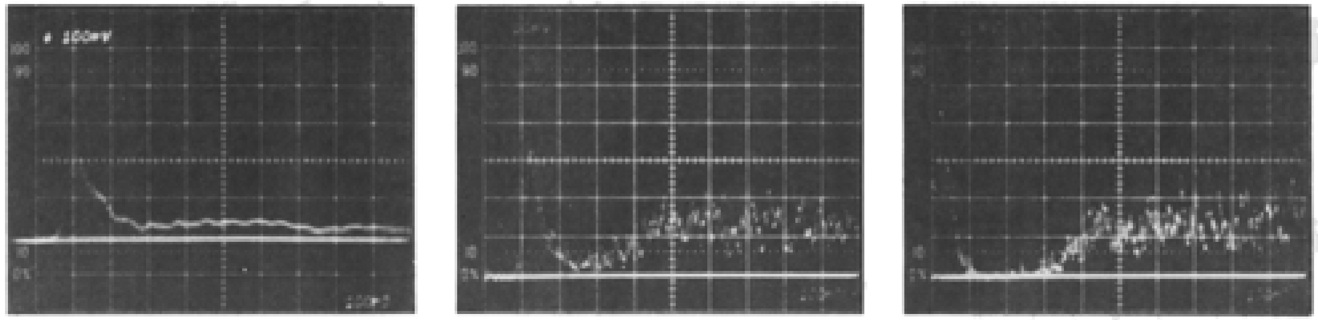
Depletion of a co-propagating Raman pump beam



ANALYTICAL MODEL OF CO-PROPAGATING RAMAN CONVERSION



Co-propagating Raman pump depletion (Los Alamos, 1974)



(a) (b) (c)

FIG. 4. Intensity waveforms for 10.6-μm pump power of 2.2 mW/cm^2; time scale 100 ns/div; time origin arbitrary in each trace. (a) Input 10.6 μm. (b) Transmitted 10.6 μm. (c) 12.2 μm spin-flip.

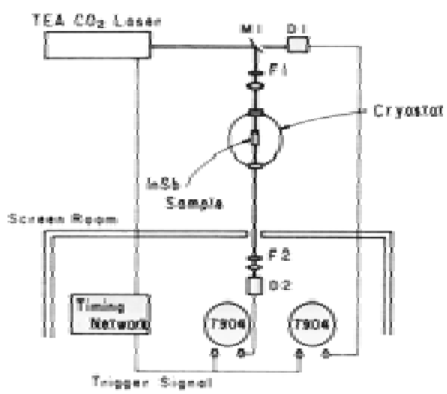


FIG. 1. Experimental apparatus.

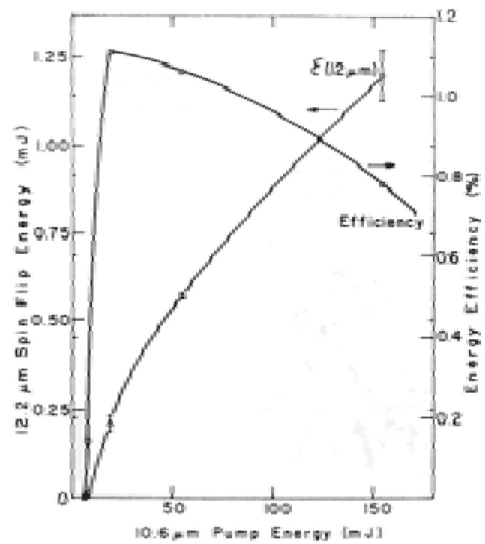
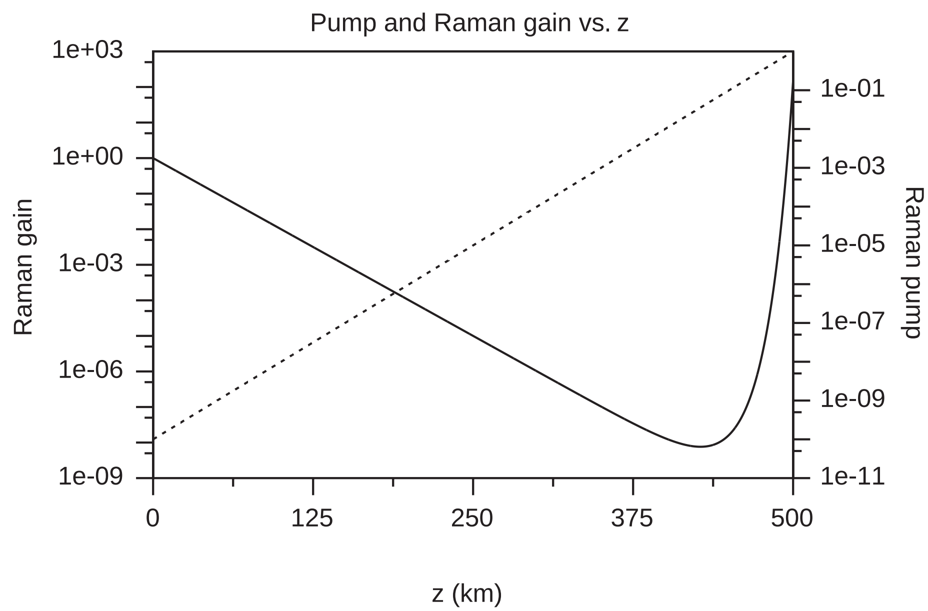


FIG. 2. Spin-flip output energy and internal conversion efficiency for various pump energies.

EFFECTIVE LENGTH AND RAMAN LENGTH

- Effective length

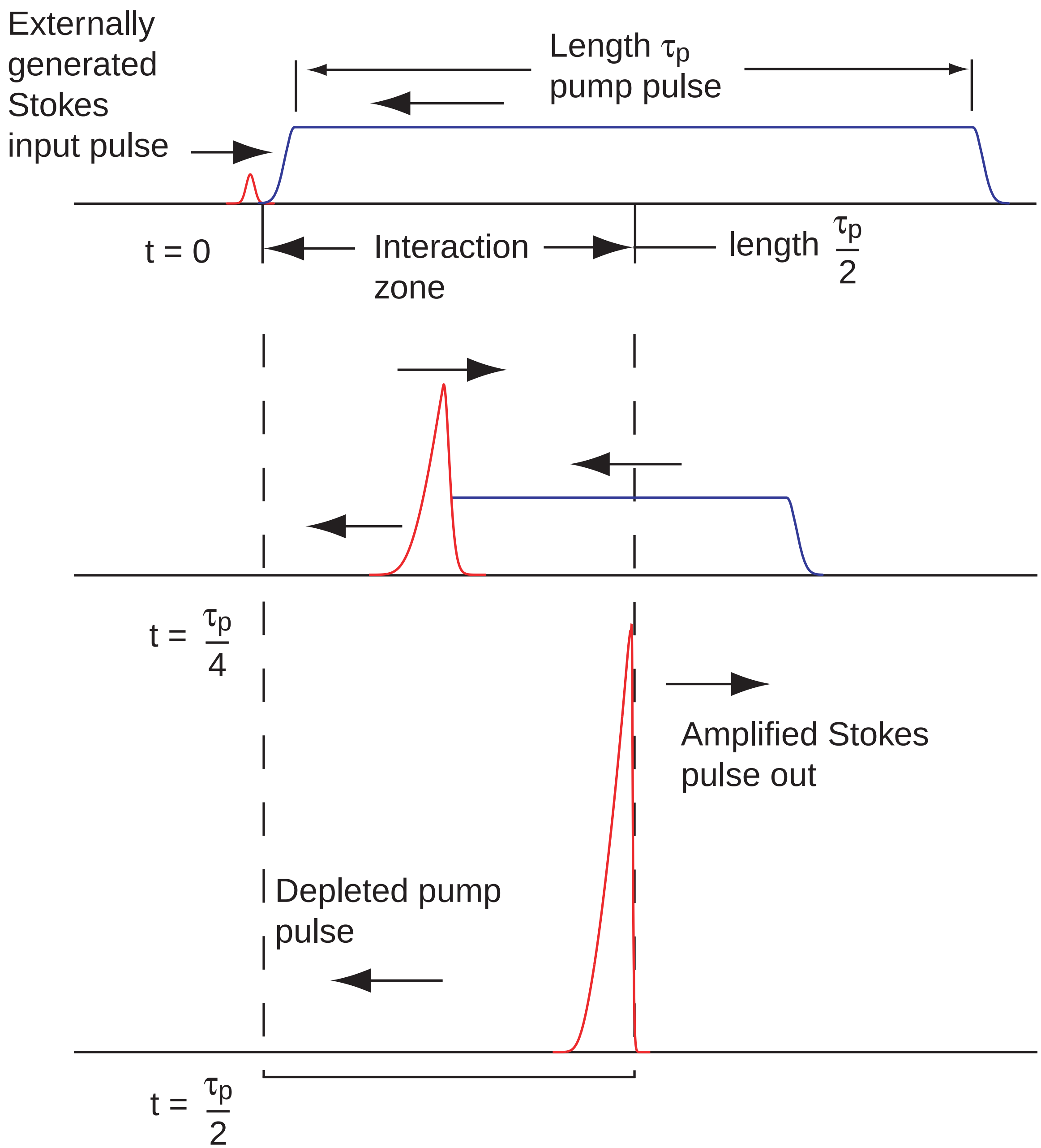
$$L_{eff}(z) = \frac{1 - e^{-\alpha z}}{\alpha} \approx \frac{1}{\alpha} \qquad \text{if} \qquad \alpha z \gg 1$$



- Raman length

$$L_R = \frac{c n_{0,L} A_e}{8\pi f_R g_R(\Delta\omega) P_L(L)} \approx 4.2 \text{ km if } P_L(L) = 1 \text{ W}.$$

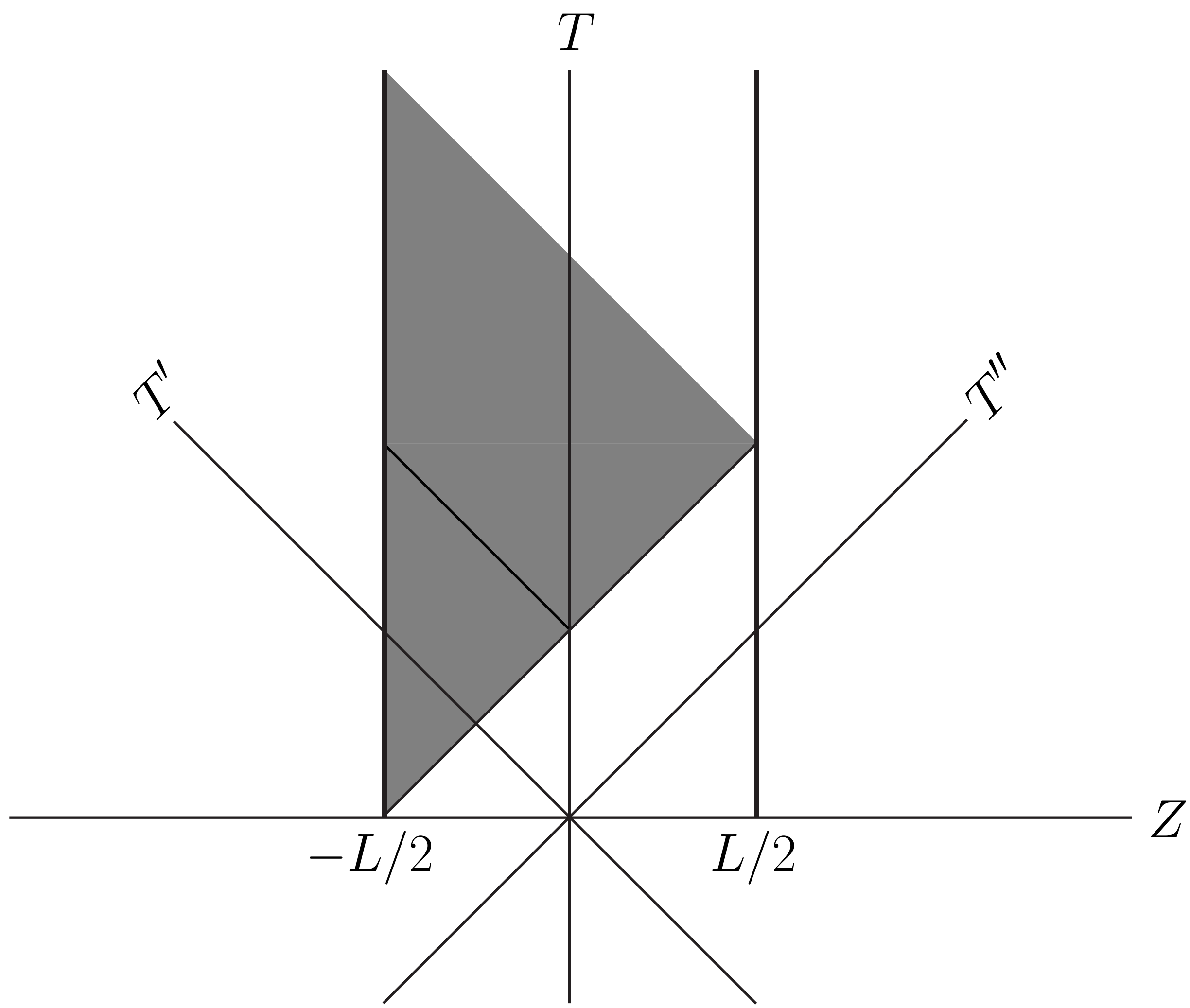
Depletion of a counter-propagating Raman pump beam



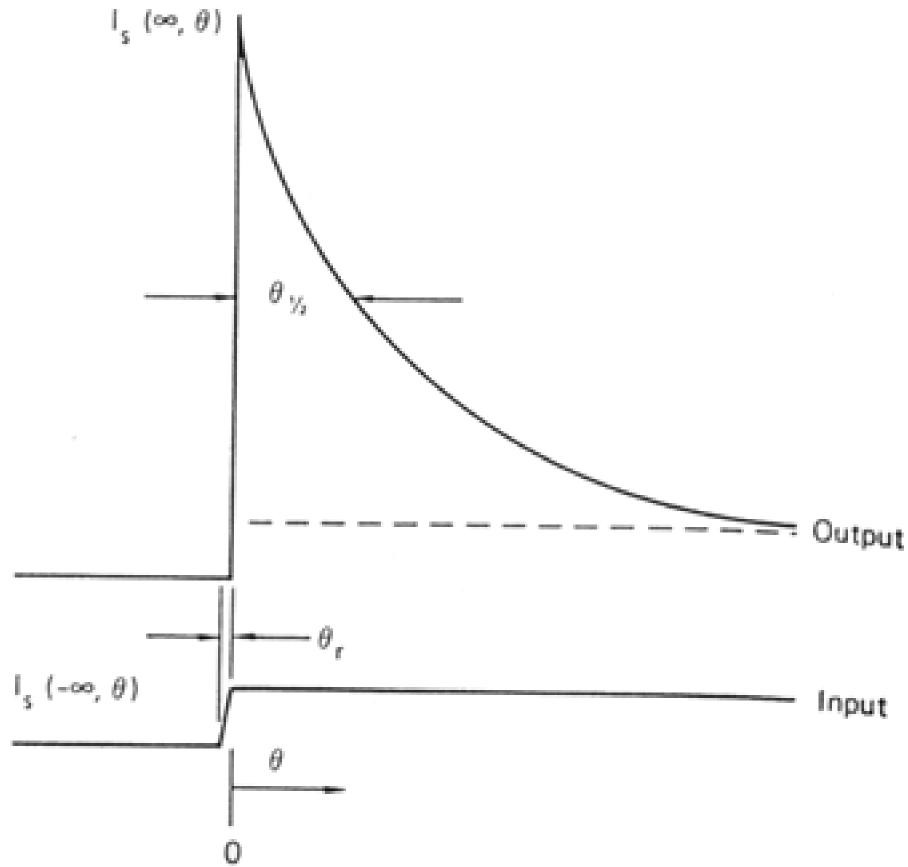
After J. R. Murray et al., JQE **15**, 342 (1979)

C. D. Cantrell and Dawn Hollenbeck 10/2001

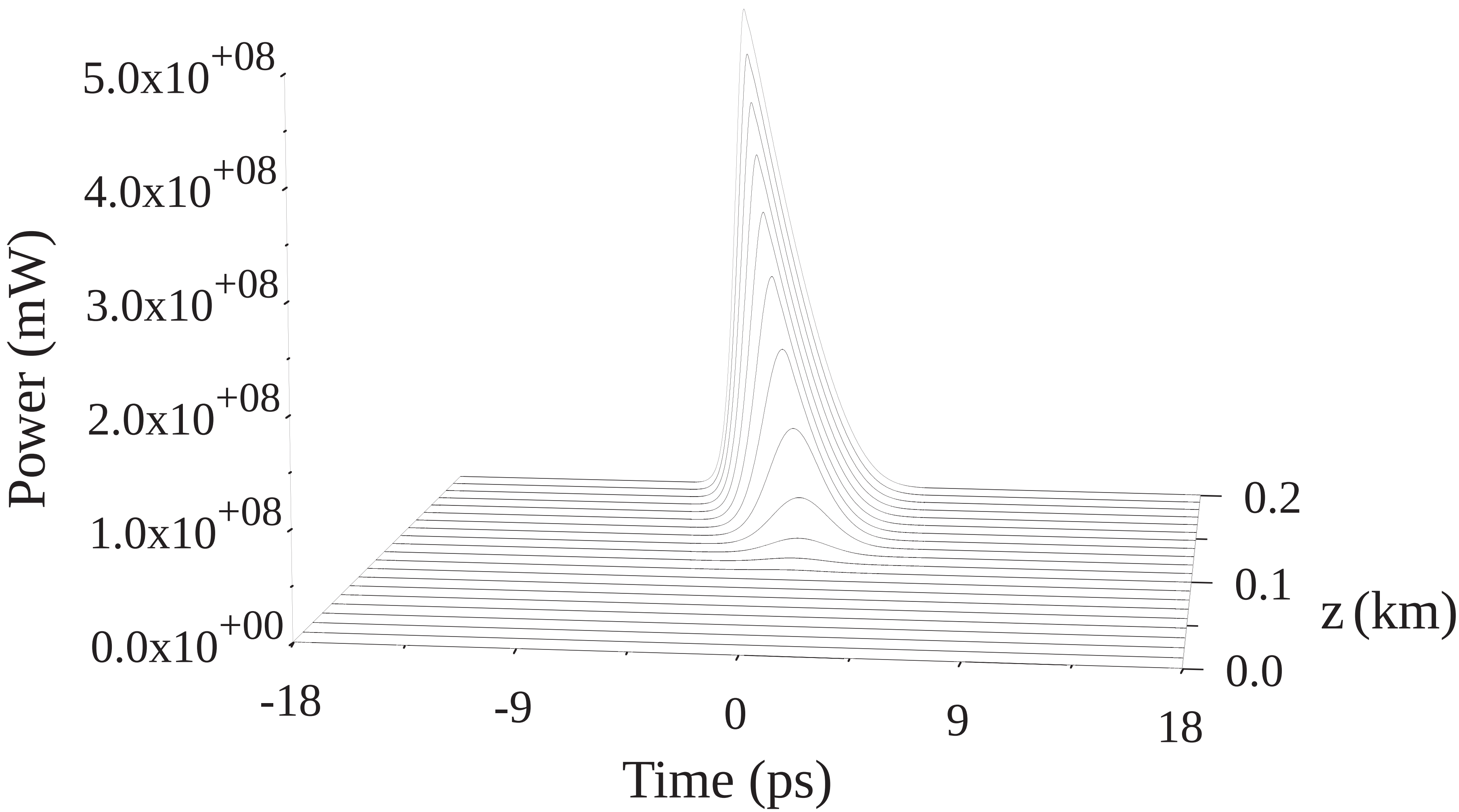
"SHADOW" CAST BY A DEPLETING PULSE



"SHARK FIN" SHAPE OF AMPLIFIED STOKES PULSE



STOKES POWER
100 W PUMP



STOKES POWER
2 W PUMP