Vector operators in the major coordinate systems

The gradient of a scalar field ψ is

$$\nabla\psi = \frac{\partial\psi}{\partial x}\hat{\mathbf{x}} + \frac{\partial\psi}{\partial y}\hat{\mathbf{y}} + \frac{\partial\psi}{\partial z}\hat{\mathbf{z}} \tag{V-1}$$

in Cartesian coordinates,

$$\nabla\psi = \frac{\partial\psi}{\partial\rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial\psi}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial\psi}{\partial z}\hat{\mathbf{z}} \tag{V-2}$$

in circular cylindrical coordinates, where $\rho = \sqrt{x^2+y^2}$, and

$$\nabla\psi = \frac{\partial\psi}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}\hat{\boldsymbol{\phi}} \tag{V-3}$$

in spherical polar coordinates, where $r = \sqrt{x^2+y^2+z^2}$.

The divergence $\nabla\cdot\mathbf{F}$ of a vector field $\mathbf{F}$ is

$$\nabla\cdot\mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \tag{V-4}$$

in Cartesian coordinates,

$$\nabla\cdot\mathbf{F} = \frac{1}{\rho}\frac{\partial(\rho F_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z} \tag{V-5}$$

in circular cylindrical coordinates, where $\rho = \sqrt{x^2+y^2}$, and

$$\nabla\cdot\mathbf{F} = \frac{1}{r^2}\frac{\partial\left(r^2F_r\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta F_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi} \tag{V-6}$$

in spherical polar coordinates, where $r = \sqrt{x^2+y^2+z^2}$.

The curl $\nabla\times\mathbf{F}$ of a vector field $\mathbf{F}$ is

$$\nabla\times\mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\mathbf{z}} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \tag{V-7}$$

in Cartesian coordinates,

$$\nabla\times\mathbf{F} = \left(\frac{1}{\rho}\frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z}\right)\hat{\boldsymbol{\rho}} + \left(\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial\rho}\right)\hat{\boldsymbol{\phi}} + \frac{1}{\rho}\left(\frac{\partial(\rho F_\phi)}{\partial\rho} - \frac{\partial F_\rho}{\partial\phi}\right)\hat{\mathbf{z}} = \frac{1}{\rho}\begin{vmatrix} \hat{\boldsymbol{\rho}} & \rho\hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ F_\rho & \rho F_\phi & F_z \end{vmatrix} \tag{V-8}$$

in circular cylindrical coordinates, where $\rho = \sqrt{x^2+y^2}$, and

$$\begin{aligned}\nabla\times\mathbf{F} &= \frac{1}{r\sin\theta}\left(\frac{\partial(\sin\theta F_\phi)}{\partial\theta} - \frac{\partial F_\theta}{\partial\phi}\right)\hat{\mathbf{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial(rF_\phi)}{\partial r}\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial(rF_\theta)}{\partial r} - \frac{\partial F_r}{\partial\theta}\right)\hat{\boldsymbol{\phi}} \\ &= \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{\mathbf{r}} & r\hat{\boldsymbol{\theta}} & r\sin\theta\,\hat{\boldsymbol{\phi}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ F_r & rF_\theta & r\sin\theta F_\phi \end{vmatrix}\end{aligned} \tag{V-9}$$

in spherical polar coordinates, where $r = \sqrt{x^2+y^2+z^2}$.

The Laplacian $\nabla^2\psi$ of a scalar field ψ is

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \tag{V-10}$$

in Cartesian coordinates,

$$\nabla^2\psi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2} \tag{V-11}$$

in circular cylindrical coordinates, where $\rho = \sqrt{x^2+y^2}$, and

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} \tag{V-12}$$

in spherical polar coordinates, where $r = \sqrt{x^2+y^2+z^2}$.

Poisson's equation is

$$\nabla^2\psi = -\frac{\rho_v}{\epsilon} \tag{V-13}$$

where ψ is the electrostatic scalar potential, ρ_v is the volume charge density, and ϵ is the permittivity.

Laplace's equation is the homogeneous (charge density = 0) case of Poisson's equation:

$$\nabla^2\psi = 0 \tag{V-14}$$

Commonly-used unit prefixes

Prefix	Meaning
f (femto)	10^{-15}
p (pico)	10^{-12}
n (nano)	10^{-9}
μ (micro)	10^{-6}
m (milli)	10^{-3}
c (centi)	10^{-2}
K (Kilo)	10^{3}
M (Mega)	10^{6}
G (Giga)	10^{9}
T (Tera)	10^{12}
P (Peta)	10^{15}